

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXV.

JANUARY 13, 1905.

No. 3

Professor H. H. TURNER, D.Sc., F.R.S., PRESIDENT, in the Chair.

Alexander John Samuel Adams, Post Office Telegraphs, London E.C., and 32 Casella Road, New Cross, S.E. ;

Capt. Arthur folliott Garrett, R.E., Craigbeg, Kingussie, Scotland ;

P. Groves-Showell, L.C.C. School for Marine Engineering, High Street, Poplar, E. ;

George Bruce Halsted, A.M., Ph.D., Kenyon College, Gambier, Ohio, U.S.A. ;

William T. Litton, "Shaftesbury" Training Ship, Grays, Essex ;

Alfred Noël Neate, C.E., 49 Fulwood Road, Aigburth, Liverpool ;

Alexander Durie Russell, B.Sc., High School, Falkirk, Scotland ;

John James Steward, F.R. Met. Soc., 457 West Strand, W.C. ;

Lewis H. Tamplin, F.R. Met. Soc., Indo-China Steam Navigation Company, Wuhu, China ; and

David Wylie, Whewell House, 9 East Road, Lancaster,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :

Brandon T. Brierley, F.G.S., Assoc. Mem. Inst. C.E., Linthwaite, Delph, Yorks (proposed by G. Calver) ;

- Maurice Farman, Observatoire de Chevreuse à Jagny, par Dampierre (Seine-et-Oise), France (proposed by Camille Flammarion) ;
Willie Venner Merrifield, B.A., Head Master, Nautical College, Byrom Street, Liverpool (proposed by W. H. Besant) ;
Isaac Molloy, M.A., Lützen, Glenageary, Kingstown, Dublin (proposed by Louis G. Macrory) ;
Alfred Edward Nicholls, Principal of King Edward VII. Nautical School, London, E. (proposed by Sir Howard Grubb) ; and
John Wearing, Garsdale, Sedbergh, Yorks (proposed by Thomas Weir).
-

Sixty-six presents were announced as having been received since the last meeting, including, amongst others :

Sir R. S. Ball, A Popular Guide to the Heavens, presented by the publishers ; Royal Observatory, Greenwich, 17 charts of the Astrographic Chart of the Heavens, presented by the Observatory ; B. Hasselberg, Untersuchungen über die Spectra der Metalle, No. VII. Spectrum des Wolframs, presented by the author ; Count de Miremont, Popular star maps ; and Practical methods in modern navigation, presented by the author ; Paris, Conférence Astrophotographique Internationale, circular, No. 11, presented by the Paris Observatory ; Science Year-book, 1905, presented by Major Baden-Powell ; Yerkes Observatory Publications, vol. ii., presented by the Observatory.

Sun-spots and Magnetic Storms. By Arthur Schuster, F.R.S.

At a recent meeting of the Royal Astronomical Society Mr. W. Maunder discussed the connexion between sun-spots and magnetic storms. The main portion of his results are embodied in a diagram which at once strikes the eye as supporting his views in a remarkable fashion. Having had, however, some experience of the manner in which the eye is frequently deceived by picking out striking features and adjusting itself so as to find what it is looking for, I approached the study of Mr. Maunder's paper in a somewhat doubting spirit, and took the trouble to form similar diagrams, using different arrangements of periods. The result has convinced me that, subject to certain qualifications, Mr. Maunder has made good his contention that magnetic storms are apt to recur in periods not differing much from that of the mean rotation of the sun-spot zones. This is a very important result, but in trying to fit a theory to suit new

facts we must not forget that older and at least equally well established ones cannot be ignored, and these do not support the theoretical views which are brought forward by Mr. Maunder.

W. G. Adams and Ellis have shown that the first shock of a magnetic storm is felt simultaneously at widely separated stations of the Earth's surface, but the more detailed examination of Adolph Schmidt* has proved that the disturbing vectors in different localities are by no means parallel to each other. Nor do they at all agree with that distribution of magnetic force which would result from the motion of electric particles projected from the Sun across the Earth's path. The disturbance in reality suggests that its cause is to be found in closed ring systems of electric currents lying parallel to, and not far above, the Earth's surface, systems which are very similar to those producing the diurnal variation of the magnetic forces. There is also other evidence tending to show that the seat of the disturbance is in our atmosphere, such as the fact, well established by Ellis, that magnetic storms occur more frequently near the equinox. In fact the whole behaviour of magnetic storms, as well as the suddenness of their first impulse, is quite inconsistent with the view that they are due to the electromagnetic effects of projected particles. We may, I think, accept it as proved that their primary cause is mainly of terrestrial origin. This view is in accord with the results of Lord Kelvin's calculation on the energy which would have to be supplied by the Sun if magnetic storms were due to electromagnetic waves emanating from that body. Lord Kelvin has never published the details of the calculation, but these are easily supplied. The outer magnetic field due to a uniformly magnetised sphere of radius a has an energy equal to $M^2/3a^3$, where M is the magnetic moment of the sphere. Such a sphere exerts its greatest force at points in the magnetic axis. Let us then take the most favourable position for the direction of magnetisation, and imagine the Sun to become a magnet with one of its poles pointing straight towards the Earth. The magnetic force H on the Earth's surface would be $2M/r^3$, if r is the distance between the centres of the two bodies. Expressing M in the terms of H , we find for the energy of the magnetic field outside the Sun's body $H^2 r^6 / 12 a^3$. Substituting numerical values ($r = 214R$; $R = .7 \times 10^{11}$; $H = .0005$) we find $.69 \times 10^{39}$ ergs for the amount of energy. In the particular storm discussed by Lord Kelvin the change of .0005 in the magnetic force took place in the course of twenty-five minutes; the average work done per second is found by dividing the total energy by 1500, thus giving $.46 \times 10^{36}$ ergs per second. This is about one-third of the value given by Lord Kelvin, but the discrepancy may easily be due to a difference in one of the suppositions made. If we had, *e.g.*, imagined the axis of solar magnetisation to stand at

* *Meteorologische Zeitschrift*, vol. xvi. p. 385 (1899).

right angles to the ecliptic the work to be done would be four times as great, and a further increase would result from taking the magnetic field within the solar surface into account.

Mr. Maunder endeavours to get over the very formidable objection which Lord Kelvin's calculation has raised against any theory which makes the Sun responsible for the energy of magnetic storms, by supposing this energy to radiate only along lines emanating from certain restricted regions of the Sun's surface. But, apart from the fact that a radiation of electromagnetic waves in restricted directions is impossible, the supposition does not help him, because the radiating surface would be diminished in the same ratio as the total amount of energy. Lord Kelvin's *reductio ad absurdum*, following from his calculation that the active portions of the Sun's surface would, in a moderate storm, emit during eight hours as much energy as the same portion of the solar surface does during four months in its regular supply of heat and light, would still remain. It may be urged that the particular theory which is apparently favoured by Mr. Maunder contemplates quite a different propagation of energy from that which underlies Lord Kelvin's calculation. That is quite true, but it is not possible to calculate the energy supply on the theory of projected particles, because that theory leaves all energy considerations out of account. If the Sun were, as supposed, sending out a continuous stream of negatively electrified particles, a highly electrified positive Sun would be left behind, and this positive electrification would continuously increase. The projection would have to take place, therefore, against strong electric forces, and work would have to be done not only in supplying the electromagnetic field, but also in overcoming electrostatic force. The theory gives no clue as to why the continuously increasing positive charge should not ultimately counterbalance the forces which tend to drive negative electrons out of the Sun. Confining ourselves to the electromagnetic field of a straight electric current, and neglecting all return currents, it is easily seen that the energy supply would have to be infinitely great. This, of course, shows that the return current cannot be neglected, and this again shows, as already stated, that the theory is not one to which in its present form energy considerations can be applied.

I am not arguing against the view that electric discharges may take place within the region of the Sun's corona, but only against the theory that such discharges can account for the energy supply of magnetic storms. It is not necessary to pursue this part of the subject further, as apart from the arguments given the theory of projected particles does not account for the electromagnetic effect observed in a magnetic storm. We may turn, therefore, to the statistical portion of Mr. Maunder's paper.

If a large number of independent events are arranged in any period whatsoever it will frequently occur that two take place near the same phase in two successive periods. But with the number of storms treated by Mr. Maunder (and considering the

length of the period) it could only happen in exceptional cases that one should take place within a few hours of the same phase in three or more successive periods. I have convinced myself of this by taking periods slightly differing from that of the solar rotation and examining their periodic recurrence. We may, therefore, accept it as proved that magnetic storms show some kind of periodicity, the length of the period being somewhere near 27·27 days. Looking at the matter without any preconceived opinions at all—and a preconceived opinion formed on previous knowledge is quite legitimate—Mr. Maunder's work does not necessarily compel us to admit that this period is due to solar influence. The sidereal and anomalistic months, for instance, have periods which are so nearly equal to that of the Sun's rotation that for a period of two or three rotations a lunar action could not be distinguished from one due to the Sun, and Mr. Maunder's results are quite consistent with the view that the active body is the Moon and not the Sun. The solar action may be more probable, owing to the undoubted connexions already established, but speaking purely of the conclusions which can logically be drawn from Mr. Maunder's statistics independently of other arguments the possibility of lunar action is equally indicated.

In order to clear up my own ideas on the point I have applied a method which I have already advocated and used in several publications. If a number of events like earthquakes or magnetic storms are arranged in any period whatsoever, we may calculate the coefficients of the periodic series which, according to Fourier, completely represents the succession of events. If the events are quite unconnected we may calculate independently the expectancy of the Fourier coefficients and the probability that any particular coefficient should exceed a given amount. If there are m events, and the value $1/m$ is given to each, so that their total sum is numerically expressed by unity, and if, further, a and b represent the two coefficients belonging respectively to the cosine and sine of any arbitrarily assumed period, the expectancy of $a^2 + b^2$ is $4/m$, and the probability that $a^2 + b^2$ should be greater than $4k/m$ is e^{-k} . The quantity $a^2 + b^2$ I call, for shortness, the intensity of the particular period. These results hold only on the supposition that all events are quite independent. If there is a real period which affects them, then for this particular period we shall, if m is sufficiently large, obtain a coefficient substantially larger than the expectancy, and the method supplies, therefore, a criterion for such a real periodicity. But in the case contemplated by Mr. Maunder no real periodicity is suggested by him, but only the probability of the recurrence of the event after a certain length of time. I foresaw this case in my first publication on the subject,* and showed that here also the squares of the

* *Terrestrial Magnetism*, vol. iii. p. 13 (1898).

Fourier coefficients would be increased by the recurrence. If magnetic storms were to occur always in groups of two, separated by a definite interval, the expectancy of the intensity would be doubled for all periods which are multiples of the interval.

In Table I. I have collected the results obtained by using Mr. Maunder's list of magnetic storms and taking the times given by him for the commencement of the storms. Arranging these times in periods, there is no difficulty in obtaining the Fourier coefficients by well known processes. The first column gives the period chosen, while the second column contains the values of the intensity belonging to this period. If we take a period equal to one-half that given in the first column we obtain the corresponding squares of amplitude as given in the third column; similarly the next three columns give the values of $a^2 + b^2$, where the period is the n th part of that given in the first column, the number being placed at the head of each column. These mean values of the different columns and rows are also given, and finally the expectancy for each period. The total number taken account of was 270, and hence the expectancy is $\frac{4}{270}$.

Table II. gives similarly the results when the period is that of the synodic revolution of the Moon. The number of storms taken into account was 278 in this case, and the corresponding expectancy is .0144. The numbers were arranged so that the whole period of the lunar month was subdivided into thirty parts. This made it easy to calculate the Fourier coefficients for seven different subdivisions of the period. The mean value of the squares of the amplitudes coincides, to the number of decimal places given, with the expectancy, thus giving a satisfactory confirmation to the treatment of the problems by the theory of probability.

TABLE I.

Period in Days.	1.	2.	3.	4.	5.	Mean.	Expectancy.
27.250	.0365	.0121	.0053	.0059	.0021	.0143	.0148
27.278	.0073	.0878	.0165	.0457	.0258	.0366	.0148
27.333	.0081	.0088	.0187	.0100	.0258	.0123	.0148
27.542	.0560	.0046	.0061	.0124	.0021	.0162	.0148
Mean	.0290	.0283	.0116	.0162	.0140	.0199	...

TABLE II.

n	$a_n^2 + b_n^2$	n	$a_n^2 + b_n^2$
1	.0177	6	.0011
2	.0231	10	.0407
3	.0053	15	.0031
5	.0098	Mean	.0144
		Expectancy	.0144

There is, then, no evidence of a period depending on the lunation, but the interpretation of Table I. is somewhat perplexing. Leaving for a moment the period of 27·278 days out of consideration, the average square of the amplitude is 0·143, which agrees closely with the expectancy calculated on the supposition that magnetic storms are distributed quite at random without any relation to solar rotation. But the period temporarily omitted alters the aspect of the question. Were we to look at this table from the purely statistical point of view we should be justified in concluding, with some degree of probability, that there is a definite period of $\frac{1}{2} \times 27\cdot278 = 13\cdot64$ days in the breaking out of magnetic storms; for the ratio of the actual to the expected intensity is 5·9, and the probability of the accidental occurrence of this ratio is three in a thousand. This, of course, is not sufficient for anything amounting to a proof, but it gives at any rate a certain presumption in favour of a real connexion. It is certainly remarkable that for four out of five of the coefficients calculated the amplitude is greater than what is to be expected on the supposition of a random distribution. A closer examination of the numbers from which the Fourier coefficients have been derived shows that the period of 13·64 days is due to the fact that during certain parts of the twenty years examined storms were apt to occur in greater quantity when the longitude of the Sun's centre was somewhere between 55° and 90° , and that at other times the occurrence predominated when the longitude of the Sun's centre differed from the above by about 180° .

According to Mr. Maunder's view that a magnetic storm, whenever it occurs, is apt to be followed by another storm at intervals of time which vary according to the particular solar latitude which determines the storm, the expectancy for the intensity of all the periods in Table I. should be raised.

The table, as far as it goes, does not support this view. Nevertheless it cannot be said that there is a definite contradiction to it. We have, in fact, the choice between two interpretations.

1. Magnetic storms are apt to occur at times which, starting from a certain point, are multiples of 13·64 days. During some years the odd multiples, and during other years the even multiples, are principally concerned.

2. Magnetic storms often recur after several successive intervals which are equal to some lapse of time sufficiently near 27·28 days to fall within the limits of rotation of the sun-spot zones.

The first alternative is that more directly indicated by the above reduction of observations. It would explain all results obtained by Mr. Maunder, but its acceptance would involve the necessity of believing in the existence of some definite period equal to 27·28 days. As different sun-spot zones rotate with different velocities this would mean either that sun-spots have nothing to do with the phenomenon or that only the sun-spot

zones having that particular time of revolution are concerned. It would also mean that the activity is to some extent concentrated on definite meridians. There are great difficulties in the way of accepting this conclusion. On the other hand the second alternative, though it presents fewer theoretical difficulties, would compel us to assume that the exceptional values of the coefficients in the period of 27·278 days is accidental. If we accept the second alternative, and take the mean of all intensities collected in Table II. as the expectancy which corresponds to periods equal approximately to the solar rotation, I calculate that the probability of an accidental occurrence of an intensity of ·0878 is ·009, or nearly one in a hundred. In twenty trials we should have, therefore, obtained a number which on the average should only occur once in a hundred trials. The probability that the average of five observations should be ·0366 is about one in twenty-one, *i.e.* in the natural course of calculation we should find that, by taking the average of five intensities, we should get one case in twenty-one where its value is equal to or greater than the number found. In our trials we have actually found one case out of four. The possibility of accident is, therefore, considerable, yet the curious fact that it is just that one period which corresponds to Carrington's adopted value for the mean solar rotation is remarkable, and invites further investigation.

It would be important to discuss in this respect the Greenwich photographs of magnetic disturbances prior to 1882. This would in all probability allow us to decide which of the two above alternative interpretations is the correct one.

The first part of our discussion has shown us that the energy of magnetic storms is of terrestrial origin; in the second we have come to the conclusion that, in some form or other, storms recur at intervals apparently identical with that of the revolution of the sun-spot zones. We must now try to reconcile the two results. It will be useful to calculate, in the first instance, the energy involved in these storms, assuming them to be due to electric currents circulating in our atmosphere. It will be allowable to consider these currents to be confined to a spherical shell having a radius equal to the Earth's radius.

For the energy of the magnetic field we have, if Ω is the magnetic potential,

$$E = \frac{1}{8\pi} \int \frac{d\Omega}{dn} dS,$$

where the integration is to be extended over the surface, n being taken in the direction of the normal drawn inwards for the outer field, and outwards for the field which lies inside the shell. The magnetic field of the disturbing currents overlaps that of the Earth, and there may occasionally be a diminution instead of an increase of energy; but, on the supposition that the two are independent, we may say that on the average there is an increase in energy equal to that required for the establishment of the dis-

turbing field taken by itself alone. We express that field in terms of its spherical harmonic components, and may therefore write outside the shell

$$= \frac{A_n a^n S_n}{r^{n+1}},$$

inside the shell

$$= \frac{A_n r^n S_n}{a^{n+1}}.$$

In these expressions a is the radius of the Earth, r the distance from the centre of the Earth, A_n a constant, and S_n a surface harmonic of degree n , being of the form

$$\cos \sigma \phi \frac{d^n P_n}{d\mu^n},$$

where P_n is the zonal harmonic of degree n and μ the cosine of the colatitude.

Substituting in the integral we easily find for the energy of the whole field

$$E = \frac{2n+1}{8\pi a^3} A_n^2 \int S_n^2 ds,$$

and finally making use of well known properties of spherical harmonics

$$E = \frac{(n+\sigma)!}{(n-\sigma)!} \frac{A_n^2}{4a}.$$

If $\sigma = 0$ the expression is to be doubled.

For the sake of simplicity consider the horizontal magnetic force at a point of the equator (referred to the axis of reference). Its maximum value lies north and south when $n-\sigma$ is odd, and east or west when $n-\sigma$ is even. Its intensity H may be expressed in the form

$$Ha^2 = kA_n,$$

where k is a numerical factor.

The energy of the field in terms of H is

$$E = \frac{(n+\sigma)!}{k^2(n-\sigma)!} \frac{a^3 H^2}{4}.$$

Table III. gives the values of $(n+\sigma)!/k^2(n-\sigma)!$ when n is three or less.

TABLE III.

n	1	2	3
1	2	...	...
2	$\frac{2}{3}$	$\frac{2}{3}$	...
3	$\frac{16}{3}$	$\frac{30}{225}$	$\frac{16}{45}$

To get an idea of the order of magnitude we may take the case $n = 1$, $\sigma = 1$, so that the energy is equal to $\frac{1}{2}a^3H^2$. Applying this to the case of the Earth, and to the same magnetic storm discussed above, in which H increased by $\cdot 0005$ in twenty-five minutes, we find

$$E = 3 \cdot 22 \times 10^{19},$$

and for the rate of doing work $2 \cdot 15 \times 10^{16}$ ergs per second, or 2.82 million horse power. The power is equal to that required to heat 7700 metric tons of water in twenty-five minutes from the freezing to the boiling point of water. It is thus seen that, though the actual force measured by our magnetographs may only be equal to the weight of one-hundredth part of a milligram, the total work done in a general storm affecting the whole Earth simultaneously is very large indeed. The large numerical value obtained limits us, as far as I can judge, to three alternative explanations of the origin of magnetic storms.

1. Our calculation of the energy required is based on the supposition that the magnetic field is independent of others on which it is superposed. But there may not be such an independent addition to the existing magnetic forces, but only a change in direction. Thus when we produce an artificial magnetic storm by moving magnetic masses in the neighbourhood of our instruments no energy is required, because the magnetic field remains the same relative to the magnet, though it is shifted in space. Similarly, in the diurnal variation of terrestrial magnetism no energy has to be supplied except that required to overcome the ohmic resistance of the conductors through which the currents are passing, because the magnetic changes are here produced by a rotation of a magnetic system which does not change relatively of itself. We might, therefore, suppose that a portion of the magnetic forces which are observed on the surface of the Earth are due to some external electric currents. As long as these rotate with the Earth there is no disturbance, but if for some reason the position of these currents relatively to the Earth were disturbed strong magnetic forces might come into play. In such a case our calculation would not apply. It is quite likely that a small portion of the normal magnetic field of the Earth is due to outside currents, but I do not consider that this portion can be sufficiently large to cause by means of its sudden variations the forces observed in magnetic storms.

2. We might imagine that the heat energy contained in our atmosphere is sufficient to supply the work done in magnetic storms. We find, indeed, that a lowering of temperature of the thousandth part of one degree throughout the mass of the atmosphere would be sufficient to supply the required work. But we must reject this theory, because only a small part of the atmosphere can be concerned in a magnetic storm, and because we do not know of any manner in which the heat of a gas can be converted into electromagnetic energy.

3. We are now left with the one remaining source of energy, which is sufficient for our purpose. That is the *vis viva* of the Earth's diurnal rotation. In C.G.S. units that energy is expressed by 26×10^{35} . That this energy, if we can find means of applying it, is capable of maintaining magnetic storms may be shown by a simple calculation. Suppose that a storm, on the average, lasts in full force forty times the twenty-five minutes we have considered in the typical case, and that we have 100 such storms in one year. The energy drawn away from the Earth would gradually diminish its rotational velocity, but so slowly that after provision has been made for the supplying of magnetic storms during a million years the Earth, as a time-keeper, would then be losing at the rate of only one second per year. Even a layer of air, at atmospheric pressure of the tenth part of a millimetre thick, has an energy of rotation more than two thousand times greater than that required for the production of a magnetic storm lasting twenty-five minutes.

Having been led to what seems to be the only available source of energy we must try to form some idea as to the manner in which the frequency of the magnetic storms may be influenced from outside, while their primary cause lies within the terrestrial atmosphere. It is important for this purpose to keep in mind that the intensity of an electric current always depends on two things—electromotive force and resistance. The electromotive force must supply the energy, but the resistance of air is affected by outside agencies, such as corpuscular emissions or ultraviolet radiation. When I discovered, in 1887, that a gas may artificially be brought into such a state that it ceases to behave as an insulator even to the smallest electromotive force, I at once pointed out the bearing of this on some phenomena of terrestrial magnetism, and suggested, in my first publication on the subject, that the greater amplitude of the diurnal variation may be accounted for by the greater conductivity of the outer regions of the Earth's atmosphere when the number of sun-spots is great.* Returning to the same subject in the concluding portion of my Presidential Address to Section A of the British Association at Edinburgh, I further suggested that both the periodicity of sun-spots and the connexion between these spots and magnetic disturbances on the Earth may be due to a periodically recurring increase in the electric conductivity of the parts of space surrounding the Sun. Since then the possibility of corpuscular projections has been discovered. Such corpuscular projections do, in passing through gases, invariably increase the electric conductivities. Ultraviolet light acts in the same way, provided it falls on sufficiently large conglomerations of matter. Without forming, therefore, any very definite theory we may accept the view that there is some solar effect propagated in straight lines which may increase the electric conductivity of the atmosphere,

* *Proc. Roy. Soc.* vol. xlii, p. 371 (1887).

and therefore set a magnetic storm going without supplying its energy.

The effects we must assume to be propagated in straight lines, because curved rays are impossible to explain unless we assume sufficient matter in interplanetary space to give it permanent conductivity. The curvature might result from an irregular distribution of such matter, and would not, in that case, revolve with the Sun, but curved rays are only required if we wish to connect directly the solar influence with sun-spots. For this supposition I can see no necessity at present.

The sudden breaking out of a magnetic storm suggests the analogy of the passage of an ordinary electric spark, which is set going by ultraviolet radiation or radioactivity. The electromotive force being supplied, no passage of electricity takes place as long as there is insufficient conductivity. A small quantity of radium brought into the neighbourhood of the electrodes may impart the necessary conducting power, and the spark then will break through the air gap.

It has been shown that the Earth's rotation must supply the energy, and this it can only do if the electric currents are induced by motion across the Earth's lines of force.

All relative motion between any extensive portion of the Earth's atmosphere and the Earth itself sets up an electromotive force which may produce currents of sufficient intensity to affect our magnetometers. The diurnal variation agrees with the effects which we should expect from the horizontal flow which must accompany the atmospheric waves which we observe directly in the diurnal variation of the barometer. Similarly the general effect of convection currents being in the direction of equalising angular momentum in different parts of the atmosphere, there must be a general drift from east to west in the equatorial, and from west to east in the polar regions. This sets up an electromotive force towards the equator in the polar regions, and downwards in the equatorial regions. Again, near the limits of our atmosphere, if there is any conducting matter not rotating with the same angular velocity as the Earth, currents will be produced, and in each of these cases a sudden variation of conducting power might set a magnetic storm going. If the resistance be diminished in any region—as, *e.g.*, along a short stretch of a meridian circle at a place where the electromotive force runs north and south—the resulting effects would be very similar to those sketched out by Dr. A. Schmidt as actually occurring in magnetic storms.

This attempt at an explanation is only brought forward in order to show that there is no impossibility of accounting for the observed disturbances in a perfectly rational manner. The theory requires, however, to be worked out quantitatively before its value can be finally tested. The difficulty begins when we wish to explain the manner in which solar phenomena affect the sequence of events. At present we are only able to form more

or less plausible guesses as regards the necessary mechanism, but the particular guess ventured upon by Mr. Maunder is not, I think, consistent with well established facts. I cannot, therefore, agree with his somewhat boastful claim that he has rendered clear what Lord Kelvin has called a "fifty years' outstanding difficulty." He has, no doubt, added a new fact and made an important contribution to the subject. He has given renewed interest to it and brought out the urgent importance of further investigation, but the mystery is left more mysterious than ever; the facts have become harder to understand and more difficult to explain.

Magnetic Storms and associated Sun-spots. By the Rev. A. L. Cortie, S.J.

The first and most important conclusion of Mr. Maunder's paper on "Magnetic Disturbances, 1882 to 1903, as recorded at the Royal Observatory, Greenwich, and their Association with Sun-spots" (*Monthly Notices, R.A.S.* vol. lxv. No. 1, 1904 November) is, that the origin of magnetic disturbances on Earth lies in the Sun, and not in any body or bodies affecting both. That is, that the Sun is the seat of the efficient cause of our magnetic storms, for evidently Mr. Maunder is not speaking of a mere condition. And the premisses for this conclusion are: first, that the magnetic storms mark out the synodical rotation-period of the Sun; and secondly, that what marks out the synodical rotation of the Sun must be caused by something in the Sun—this second premiss is implied, but not expressed, and is not necessarily true—therefore the cause of magnetic storms lies in the Sun. But the argument of Mr. Maunder is faulty, in that it omits at least one other alternative: namely, that both the magnetic storms on Earth, and the magnetic centres on the Sun associated with sun-spots, which are presented to the Earth at successive synodical rotations, may have a common cause which is neither on the Sun nor on the Earth. This latter alternative is consonant with the theory proposed by Father Sidgreaves in his memoir "On the Connexion between Solar Spots and Earth Magnetic Storms" (*Memoirs, R.A.S.* vol. liv. p. 91). For when two sets of connected phenomena are found regularly to concur, the logical conclusion is not necessarily that one is the cause of the other, but that either one is the cause of the other, or they have a common cause. It is this latter alternative of which Mr. Maunder has omitted to take account, and which renders his syllogism out of form. Therefore, even were he to prove his first premiss—namely, that magnetic storms mark out the synodic rotation-period of the Sun—up to the hilt, it would still not

follow logically that the cause of magnetic storms resides in the Sun.

There is no doubt that some positions of the Sun relatively to the Earth are more favourable conditions, and perhaps even necessary conditions of movements of the magnets, the "diurnal range" of the declination magnet, the "annual inequality," the greater prevalence of magnetic storms at the equinoxes, are abundant evidence of this. Nor, again, is there any doubt of the close connexion of magnetic storms with the spotted area and number of spots on the Sun. The laborious memoir of Father Sidgreaves (*loc. cit.*) and the papers of the Greenwich observers have shown this to be true, taking the spots in detail. This evidence is in addition to the statistical work of Mr. Ellis, which demonstrated the close accord of the variations of diurnal range of the declination and horizontal force magnets over a long period of years, with the curve of annual relative sun-spot numbers as prepared by Professors Wolf and Wolfer. But the same examination in detail of sun-spots and magnetic storms, made by Father Sidgreaves for the period 1881-1898, and by the writer for the minimum years 1899-1901 (*Astrophysical Journal*, vol. xvi. No. 4, 1902 November), has shown glaring exceptions to the general agreement of spots and magnetic storms, which precludes the possibility of the one being the cause of the other. If Mr. Maunder could completely prove his first premiss—and he only attempts to do so for at the most about 75 per cent. of the magnetic storms tabulated—the only conclusion that could be drawn would be, that the synodical rotation period of the Sun for middle latitudes, when sun-spots are present on his disc, is a favourable and perhaps necessary condition for magnetic storms on Earth. Professor Schuster has proved (*Report of the British Association*, Belfast, 1902, p. 518) that "the magnetic declination at Greenwich shows no period between 25.5 and 27.5 days having an amplitude as great as 6'," and Dr. Chree has shown (*Proc. R.S.*, vol. lxxi. p. 224) that Wolf and Wolfer's provisional curve of relative sun-spot numbers are matched by the curves of range of the magnetic elements, whether the sun-spot frequency curves be plotted for the magnetically "quiet" days only, or for all days, whether magnetically quiet or active. Therefore, so far as the diurnal range of the declination magnet at least is concerned—for Schuster's results deal only with this quantity—it follows that the synodical rotation period of the Sun does not affect the magnetic curve, and, on the other hand, that the curve of sun-spot frequency is independent of magnetic storms. But if Mr. Maunder's contention is correct, that greater sun-spots mark magnetic centres of activity on the Sun, which are so effective, and almost exclusively so on the Earth, that magnetic storms mark out the synodical rotation period of the Sun, then, on the one hand, the synodical rotation period of the Sun ought to be reflected in the curve of diurnal range of the declination magnet, which it is not, and, on the other

hand, the curve of relative sun-spot frequency should not be almost identical for "quiet" and all magnetic days, which it is.

To come now to Mr. Maunder's major premiss, that magnetic storms mark out the synodic rotation period of the Sun, in at least fifty per cent. of the cases observed. The chief evidence for this statement is set forth in Table III. of his paper; and as it is impossible at present to examine every sequence set down in the table, it is proposed to take the first and the second, and incidentally the fourth and the twenty-third, as also the thirty-second, for more detailed criticism. This last sequence is selected because it is claimed, in the first place, that it connects six storms which occurred in successive synodical rotations of the Sun, and in the second place because it includes two magnetic storms which, as Father Sidgreaves pointed out occurred at an epoch of minimum solar activity, when the Sun had been almost absolutely free from spots and faculæ for a considerable period. Table A of Father Sidgreaves' paper (*loc. cit.* p. 93) gives an example of the method in which the relation of sun-spots and magnetic storms was studied at Stonyhurst; and if the various sequences of Mr. Maunder's paper are studied in relation to the spots with which they were presumably connected, it will appear doubtful, whether the sequences in many cases might not be purely fictitious, so far at least as they are associated with sun-spots.

The first sequence in Mr. Maunder's list is connected with groups 53 and 53a of the Stonyhurst series, the life-history of which is given in the annexed chart. It is well to state here that the materials which have been utilised in the present discussion were prepared by Father Sidgreaves for his memoir. A summary history of these groups is also to be found in my paper on "The Duration of the Greater Sun-spot Disturbances for the Years 1881-99" (*Monthly Notices, R.A.S.* vol. lx. No. 8), the numbers of the groups here discussed tallying with those in Table I. of that paper.

These two associated groups, one being the recurrence of the other, persisted for five solar rotations. During the first rotation there were five days which were magnetically disturbed, the first storm occurring when the spot had just passed the central meridian. Four more days of magnetic disturbance occurred while the spot-group was on the invisible hemisphere of the Sun. During the second rotation there were only two magnetic storms: one did not coincide with any of the five days of magnetic disturbance of the first rotation, the other did. Surely such a coincidence is merely accidental. However, there is this fact in favour of a real synodical rotation coincidence, that the two storms that did coincide were the greatest of the whole series. Let it be supposed then that these two storms, one of the first and the other of the second rotation, mark out a magnetically active region of the Sun associated with the sun-spot. The sun-spot group returned

CHART I.

Sun-spot Group 53 and Associated Magnetic Disturbances. Sequence I.

M = Magnetic Storm Magnitude. S = Spot Magnitude.

C = Central Meridian. Ci = Central Meridian on Invisible Side. E, W = East and West Limbs.

	Ci.	E.	C.	W.	Ci.	Spot Number. Stony-Green- hurst. wch.
^{1892.} July	[28] 29 30.	1 2 3 4 5 6 7 8 9 10 11. 12 13 14 15 16 17 18 19 20 21 22 23			July	53 2581.
M.			3 4 5.	11.* 8	3	
S.		13 16 18 24 22 20 15. 15 11 13 4				
July	24 25 26 27 28 29 30 31.	1 2 3 4 5 6 7 8 9 10 11 12 13. 14 15 16 17. 18			Aug.	53 2611.
M.	3 3 3		3	9*		
S.		3 12 10 8 9 9 7 5 5 4 5 4 4				
Aug.	19 20 21 22 23 24 25 26 27 28 29 30 31	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31			Sept.	53. 2643
M.						
S.		2 2 3 3 3 2 2 2 2 2 3 2 2 2 1				
Sept.	15. 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30			Oct.	53. 2675
M.	4 4				3 3	
S.		1 1 1 1 1 1 2 3 2 2 1 1 1				
Oct.	13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31			Nov.	53a 2718.
M.	3 4 3 *3 3				6	
S.			1 1 0'4 0'1			

* Reference numbers of disturbances: Maunder, 151, 154, 156.

Mean heliographic coordinates, Group 53 ; long. 91°, lat. +11°.
53a; long. 94°, lat. +11°.

Spot-group probably born on June 28.

CHART II.

Sun-spot Group 29 and Associated Magnetic Disturbances. Sequences II., IV., and beginning of XXIII.

	El.												C.												W.												Cl.												Spot Number. Stony- Green- hurst. with.
	Ci.												C.												W.												Cl.												
1886.																																																	29. 1861
Mar.																																																	
M.																																																	
S.																																																	
Apr.																																																	
M.																																																	
S.																																																	
May																																																	
M																																																	
S.																																																	
June																																																	
M.																																																	
S.																																																	
July																																																	
M.																																																	
S.																																																	
Aug.																																																	
M.																																																	
S.																																																	
Aug.																																																	
M.																																																	
S.																																																	
Sept.																																																	
1924																																																	
1925																																																	

Reference numbers of disturbances: Maunder *, 72, 73, 74. † Numbers 69, 70. § 75. Mean heliographic coordinates, long. 71°, lat. -10°.

for three more rotations, diminished in size it is true, and the magnetic centre ceased to act. And yet in other cases, notably in those of 1889 November, we are to suppose, in order to form a sequence, that a magnetic centre can be active, not only during the life-history of a sun-spot group, but long after all spots to mark the magnetic centre of storms have disappeared. This is anomalous. Moreover, though this particular sun-spot group was diminished in size, it could still be magnetically active, on the supposition that these sequences of magnetic storms were connected with the sun-spot, for in the fourth rotation, when it came to the E. limb, another active storm occurred (omitted from Mr. Maunder's list), which forms an equally good sequence with No. 156 of his Table I.—which occurred at the fifth appearance of the region of the spot on the E. limb—as is formed by numbers 151 and 154, marking the W. limb position. On the supposition of a stream-like set of particles emanating from the spot and impinging on the Earth to cause sequences of magnetic storms, it is difficult to imagine how both an E. limb and a W. limb, as well as a central, position of the spot, could be effective. This second sequence, too, seems to accentuate the non-activity of the region of the first sequence during the third and subsequent rotations. It fact it would seem that the magnetic storms set down in the chart have very little connexion with the spot. The most active period of the life-history of the spot, too, is marked by a storm which does not recur.

The next spot-group dealt with is that numbered 29 in the Stonyhurst lists, the details of its life-history and the associated magnetic storms being set down in the subjoined chart.

The visible duration of this sun-spot group was 170 days, and it passed the central meridian seven times. Appearing on the E. limb on 1886 March 29 as a small insignificant group, its first appearance was announced by the greatest magnetic storm of any that occurred during its life-history, and, indeed, the greatest magnetic storm of the year. The storm began with the sharp movements in the magnets characteristic of many great storms. Its number in Mr. Maunder's list is 68, and it is not a member of any of the sequences in Tables III. and VI. The spot, the advent of which it marked upon the disc, came round five more times, and this presumed focus of violent magnetic action which affected the Earth at the first rotation fails to affect it at the five succeeding rotations. But when at the seventh rotation two groups of very small spots marked the position of the former group, the region again becomes magnetically active and gives the first storm of Mr. Maunder's sequence XXIII.—the connexion being adverted to in Table VIII. of Mr. Maunder's paper—the other three members occurring at a time when solar activity, as shown by spots and faculæ, had suffered a marked diminution. This mode of action is again anomalous if sun-spots fix the magnetically active regions of the Sun. For it is difficult to explain why a violently active magnetic region

should have been quiescent for five rotations, to break out again when all spots had disappeared. Mr. Maunder in his paper appeals to intermittent action both of spots and magnetic storms, and gives examples of the one in Tables IV. and V., and of the other in Table VII. This appeal to intermittent action would have been stronger had he been able to show that the intermittent magnetic storms were associated with intermittent appearances of spots. The life-history chart of the present solar disturbance contains two other sequences, viz. II. and IV., of Mr. Maunder's paper. Sequence II. is marked by three storms, according to his enumeration, but by two only according to the Stonyhurst lists, which are determined by W. limb position of the spot. Here again one sequence marks the W. limb position, and another the E. limb position of the same spot region, and the same difficulty occurs, as was mentioned above, of a stream of particles in practically parallel lines, derived from the same active magnetic region, reaching the Earth when the spot was both on the E. and W. limbs. Sequence IV., also of two members, occurred when the sun-spot group 29 was on the invisible disc of the Sun, but the first member of the sequence is accredited to a sun-spot group, No. 28 of the Stonyhurst lists, which was visible for six successive rotations, and the second member probably belonged to a different sun-spot group altogether, No. 30 of our lists, which was visible for three rotations. Hence this concurrence to form a sequence seems to be fortuitous. There is also in this chart of group 29 a sequence of three magnetic storms which are not in Mr. Maunder's lists, and which are unconnected with any of the greater sun-spot groups of the year 1886. The result again of the study of the sequences associated with the life-history of this group serves to show, that although sequences undoubtedly exist, the nature of their connexion with the spot-groups, which are presumed to be indices of the regions of magnetic activity on the Sun, is not that of streams of particles of restricted diameter.

The sequence numbered XXXII. in Mr. Maunder's table is important, first, because it contains six members, and secondly, because it occurred at a time of minimum activity. By this sequence Mr. Maunder endeavours to connect the active magnetic storms of 1889 November with the second rotational appearance of the biggest spot of the year which was first seen on June 16. But in his table on p. 21 the longitude of the centre of the Sun's disc has altered from $57^{\circ}0$ at the beginning of the sequence to $108^{\circ}8$ at the end of the sequence. This in itself is internal evidence that the members of the sequence are only fortuitously connected, and do not mark the same magnetic region on the Sun. On pp. 29 and 30 Mr. Maunder discusses the life-history of the groups on the Sun when the first of these disturbances appeared. But besides the two groups of spots he mentions in longitude 35° , latitude -7° , and longitude 82° and

latitude -8° respectively, there was a third group, Stonyhurst number 42, which was on the central meridian four times, the Greenwich numbers of the spots being 2097, 2100, and 2102, 2103, 2107 for each successive rotation. It is with this spot-group that the storm of October 5—the fourth of the sequence—was certainly connected, the mean heliographic coordinates of the group being longitude 156° , latitude -22° . There is also quite a possibility of the second and third members of the sequence having belonged to the same spot-group. However this may be, the attempt to connect the November magnetic storm with the one great spot outburst of the year breaks down. The spots are separated 120° in longitude, and do not mark the same magnetic centre. In the Stonyhurst lists the several storms of this sequence are accredited to three spot-groups, No. 40, long. 35° , lat. -7° , No. 41, long. 82° , lat. -8° , No. 42, long. 156° , lat. -22° . The progressive longitudes of these three groups must be noted, which explains the similar progressive longitude of the centre of the Sun's disc for the various members of sequence XXXII., and therefore shows that they belong to different sun-spots. Mr. Maunder has failed to catalogue an active magnetic storm of 1889 November 17, which would give a storm occurring between the fifth and sixth members of the group when the same meridian was turned directly away from the Earth. It makes the sequence less strong than it would otherwise have been. Again with regard to this particular sequence Mr. Maunder claims (*loc. cit.* p. 23) that it is one of those that indicate a much more rapid rotation of a solar zone of latitude than the mean period 25.38 days, and it does so because the sequence marks a latitude of magnetic activity in the Sun which is near the solar equator. This point he alludes to as "exceedingly important." If Mr. Maunder could prove that the sequence was formed of members that marked a magnetically active region connected with the June spot of 1889 in latitude -7° it would go far to establish his point. But in the sequence the October magnetic storm is certainly connected with the spot in latitude -22° , and the two November storms form a sequence with this storm, and only casually with the other members of the group. Now the rotation period of latitude -22° is much slower than that of latitude -7° . It follows, therefore, that the rotation period derived from the series of six magnetic storms, by a mean which includes storms in relatively high and low latitudes, is fictitious, and is not related to the corresponding equatorial zone of sun-spots.

As far, then, as this detailed examination of the sequences connected with the synodic rotation period of the Sun has gone, it shows that the sequences in several cases are real, but that the association of sun-spots with the presumed magnetic centres giving rise to the several sequences is not a very close one. But if it be claimed that the association with sun-spots is a close one, then the mode of action of the magnetic centres by means of

streams of particles, practically parallel, and of relatively small diameter impinging on the Earth is negatived. Such a stream of particles could not be effective at positions so remote from each other as the E. and W. limbs of the Sun.

*Stonyhurst College Observatory,
January 1905.*

Observations of the Spectra of Sun-spots, Regions C to D.
By A. Fowler.

Introductory.

The observations of the spectra of sun-spots forming the subject of the present paper were made somewhat irregularly during the period 1903 October 15 to 1904 December 31. Detailed observations of widened lines were made on twenty-three separate days, and fifty-three observations of the appearance of the C line in and near spots were made on thirty-five days. The observations were restricted almost exclusively to the red end of the spectrum between C and D₂, and as many as possible of the affected lines in this region were recorded. They are therefore comparable with the observations made by Father Cortie at Stonyhurst,* and the results are of some interest as showing the degree of agreement between two independent observers working on essentially the same plan. It is possible also that some points in the discussion of the observations which I have attempted may be suggestive to other observers.

Mode of Observation.

All the observations were made with an "Evershed" two-prism solar spectroscope by Hilger, attached to the 6-inch Troughton equatorial provided for the instruction of students at the Royal College of Science, South Kensington. The definition of the spectroscope is remarkably good, and the dispersion is adequate for the identification of most of the lines. There are of course many close doubles which cannot be clearly resolved with this equipment; but if such were widened the affected component could often be judged by noting on which side the widening seemed to lie; in other cases it remains doubtful which of the components was affected.

The approximate positions of the lines were read off from Rowland's map in the usual manner, and were afterwards corrected to two places of decimals by reference to the tables of solar lines.

* *Mem. R.A.S.* vol. 1. pp. 30-56; *Monthly Notices*, vol. lxiii. pp. 468-480. Summarised in *Astrophys. Jour.* vol. xx. pp. 253-265.

Some difficulty was found at first with regard to a suitable scale by which to indicate the amounts of widening of the lines. Father Cortie's method is to state the extent of widening in tenths of the normal width of the corresponding solar line, but I found it extremely difficult to assign numbers on this plan to lines which are very feeble in the Fraunhofer spectrum ; there is, moreover, the objection that the unit of widening is different for different lines. I therefore decided provisionally on a simple scale ranging from 5 for the lines most obviously widened and darkened to 1 for those in which the widening could only just be recognised with certainty, hoping that the discussion of the results would eventually suggest some better system. The numbers assigned include the combined effects of widening and darkening and directly indicate little more than the relative ease of detection. Towards the end of the series of observations, as explained later on, it was concluded that the most useful method of recording the lines is to note the actual intensities of the spot lines by comparison with neighbouring solar lines outside the spot spectrum.

It only rarely happened that the whole of the region C to D could be minutely examined in a single observation, so that usually only the principal lines throughout this part of the spectrum were noted, or a smaller region explored more completely. By varying the latter from time to time, a fairly representative table of spot lines for the period covered by the observations has been secured, but the record may be far from perfect as regards the less affected lines. The region C to D, comprising 1700 lines in Rowland's tables, is in fact much too extensive for a single observer to explore fully ; but if the observations had been restricted to a much smaller region, any attempted generalisations would have been less satisfactory.

Description of Table.

The lines most affected appear to have been identical in all the spots examined ; and, in accordance with the results of other observers, many of these were found to be due to vanadium and titanium. The daily records differ somewhat as regards the less affected lines ; but, taking account of the difficulties of observation and the differences of observing conditions, it is possible that these apparent variations from spot to spot were chiefly due to the imperfections and incompleteness of the records. Moreover, the period of observation was too short to be likely to exhibit any marked changes depending upon the sun-spot period, such as that discovered by Sir Norman Lockyer and confirmed, in part, by Father Cortie. It therefore seems unnecessary to give all the details of the individual observations, and the results are accordingly summarised in the accompanying table.

It is scarcely necessary to point out that the widened lines do not constitute the whole of the spot spectrum ; all the Fraun-

hofer lines are usually present, and there are, in addition, "spot bands," some of which were also observed. There is also the "continuous" absorption, but the dispersion employed was insufficient to show the breaking up of this into closely adjacent fine lines which has been noted by Young and Dunér.

The first column of the table gives the wave-lengths of the affected lines, as taken from Rowland's tables.

The origins of the lines are indicated in the second column, these, with few exceptions, being as assigned by Rowland. Photographs of the spectra of vanadium and titanium which I have taken on plates stained with pinachrome show a few additional lines of these elements; but as the wave-lengths need confirmation with higher dispersion, possible origins depending upon them are noted in the column of Remarks. Apparent coincidences with telluric lines are not included as origins, but are also indicated in the column of Remarks.

Column 3 shows the number of times each line was observed. The greatest number is eighteen; but, considering the nature of the records, it is not to be supposed that the lines noted on fewer occasions were necessarily not affected when not recorded.

Column 4 indicates the mean "widening" on the arbitrary scale which has already been mentioned. The numbers derived directly from the observations have been multiplied by 2, in order to reduce them to the more convenient scale which gives 10 for the maximum.

Column 5 gives the ordinary intensities of the lines in the Fraunhofer spectrum according to the estimates of Rowland. A line of intensity 1 is clearly visible on the map; and below this the lines, in order of faintness, proceed from 0 to 0000, indicating lines more and more difficult to see.

Besides the references to coincidences with telluric lines and probable coincidences with previously unrecorded lines of vanadium and titanium, column 6 contains a few general remarks, and also indicates by a † the lines which do not appear in Father Cortie's summary of the Stonyhurst observations. Closely adjacent lines, observed by Cortie in some cases, are indicated by a wave-length following the †.

TABLE I.
Widened Lines between D and C.

Wave-lengths.	Probable Origins.	Number of Observations.	Relative Mean Widening.	Intensity in Sun.	Remarks.
5890.19	Na	10	7	30	D ₂
95.16	...	4	3	0	A wv *
96.16	Na	10	7	20	D ₁
98.38	...	2	4	4	A wv *
99.52	Ti	8	6	1	

* Indicates that the line is not in Cortie's table.

Wave-lengths.	Probable Origins.	Number of Observations.	Relative Mean Widen- ing.	Intensity in Sun.	Remarks.
5900.14	...	} 1	5	{ 2	A wv
00.26	...			{ 4	A wv
03.75	...	8	5	1	A wv *
11.37	...	1	7	0000	*
13.21	...	1	4	3	A wv *
15.65	...	4	4	1	A wv
16.48	Fe	8	5	3	*
18.77	Ti	12	7	0	* (18.64 A wv)
22.33	Ti	10	5	0	* (22.74 A wv)
23.87	...	1	6	1	A wv *
32.31	...	1	4	5	A wv
38.27	...	16	7	0	A wv. Possibly Ti
41.99	Ti	8	8	00	* (41.85 A wv)
44.95	...	7	6	1	A wv
49.57	Fe	3	6	1	
53.39	Ti	5	7	1	
55.17	...	3	3	1	A wv *
56.92	Fe	3	7	4	
60.20	...	2	3	0	A ? *
66.06	Ti	15	7	2	
71.56	...	4	5	1	
78.77	Ti	15	8	1	
85.44	...	1	6	0	A wv *
88.79	...	4	3	0	A wv *
92.22	...	5	5	00	A wv *
99.92	Ti	16	6	0	" Always much widened " (Cortie)
6005.77	Fe	11	5	1	
08.19	Fe	1	4	4	
12.45	Ni	7	5	1	
13.72	Mn	1	4	6	
16.86	Mn	2	4	6	
18.52	...	1	4	0	
22.02	Mn	1	4	6	
39.95	V	16	9	0	" Always much widened " (Cortie)
46.25	...	2	4	0	*
53.91	Ni	4	6	0	

* Indicates that the line is not in Cortie's table.

Wave-lengths.	Probable Origins.	Number of Observations.	Relative Mean Widening.	Intensity in Sun.	Remarks.
6054.29	...	2	5	00	A ? *
57.48	...	1	4	00	
58.39	...	5	6	000	* Possibly V
63.08	...	15	6	0	" Much widened always (Cortie)
64.85	Ti	17	7	00	
81.67	V	18	7	0	
82.93	Fe	1	4	.1	
84.33	...	1	4	0	*
85.47	Ti, Fe	18	6	2	
86.50	Ni	1	4	1	
88.05	...	2	4	00	
90.43	V	15	6	2	} See Note (1)
91.40	Ti	2	4	0	
93.86	Fe	1	2	2	
98.47	...	2	3	0	
6100.49	...	1	4	00	
02.39	Fe	} 7	6	2	} This group appears to be usually strengthened in spots. The Ca line is probably the most widened
02.94	Ca			9	
03.40	Fe			4	
11.87	V	16	8	0	* (11.29 Ni)
19.74	V	17	8	1	
19.97	Ni	1	4	0	
22.43	Ca	5	5	10	
26.44	Ti	16	7	1	
29.19	Ni	1	4	1	
35.58	V	15	7	00	Spot line possibly includes Cr 6135.99
45.23	...	1	4	2	
50.36	V	15	7	0	
54.44	Na	15	6	2	
56.24	...	1	4	00	
60.96	Na	} 3	6	3	
61.50	Ca			4	
66.65	Ca	3	2	5	
69.25	Ca	1	4	6	
69.78	Ca	2	5	7	
88.21	Fe	1	4	4	

* Indicates that the line is not in Cortie's table.

Wave-lengths.	Probable Origins.	Number of Observations.	Relative Mean Widen- ing.	Intensity in Sun.	Remarks.
6189.20	Co	1	4	00	A wv? *
99.40	V	17	9	0	
6210.90	...	17	7	00	Possibly Scandium. See Note (2)
12.28	...	2	2	00	
14.08	V	2	5	000	*
15.36	Fe	} 1	6	{ 5	*
15.63	Ti			{ 000	
16.57	V	14	6	1	Attributed to V by Young and seen in new photographs of V
21.01	Fe	5	5	0	Possibly Ti
21.55	Fe	3	5	00	
24.72	V	11	5	000	
29.44	Fe	4	5	1	
30.94	V, Fe	1	4	8	Strong compound line
32.86	Fe	1	4	3	
33.08	...	1	4	0000	* Possibly V
40.17	...	4	5	00	
43.06	...	} 18	10	{ 000	} "Most widened of all the lines" (Cortie). See Note (3)
43.32	V			{ 1	
49.86	...	2	4	000	*
52.05	V	12	5	00	
56.57	Ni, Fe	1	6	6	
58.32	Ti	} 8	6	{ 2	{ 0000
58.57	V			{ 0000	
58.93	Ti	4	5	3	
59.80	...	1	2	00	
61.32	Ti	} 12	5	{ 1	{ 0000
61.50	V			{ 0000	
66.55	V	5	4	000	Attributed to V by Mitchell and seen in new photographs of V
69.08	V	13	6	000	
71.49	Fe	2	2	0	
74.87	...	16	7	00	A wv. Possibly V
79.08	...	3	5	2	A (O)
80.83	Fe	3	5	3	
85.38	V	9	6	00	? *
86.36	...	2	4	0	

* Indicates that the line is not in Cortie's table.

Wave-lengths.	Probable Origins.	Number of Observations.	Relative Mean Widen- ing.	Intensity in Sun.	Remarks.
6287.95	...	1	4	1	A (O)
89.61	...	1	5	1	A (O) *
93.03	V	11	7	000	? * (92.38?)
96.58	V	12	7	0000	? * (96.17 A [O]?)
6301.72	Fe	1	6	7	
04.55	...	1	3	000	*
06.02	...	16	9	2	A (O) "Generally much widened" (Cortie). Possibly due to Scandium. See Note (2)
18.24	Fe	1	5	6	
27.82	Ni	1	2	2	
30.32	Cr	13	6	1	This is the strongest red line of Cr
62.56	Zn	1	6	1	Said to be variable in Sun
63.09	Cr, Fe	7	6	2	
64.92	...	1	2	0	
66.56	Ti	6	6	{ 000	
66.71	Ni			{ 0	
81.6	...	5	...	...	"Spot band"
89.0	...	5	...	...	"Spot band"
92.75	...	3	3	0	
6400.22	Fe	1	4	8	
05.98	...	3	4	00	
13.80	...	4	5	0000	? *
25.08	...	1	6	00	
35.26	...	2	4	0000	*
39.29	Ca	8	6	8	
50.03	Ca	8	6	6	
52.54	...	3	5	00	
55.23	Co	5	5	0	
55.82	Ca	8	7	2	
62.78	Ca	9	6	{ 5	
62.97	Fe			{ 3	
63.97	...	1	4	0000	A ? * (63.72?)
64.90	...	6	5	00	
71.89	Ca	12	6	5	
91.88	Mn	1	4	000	

* Indicates that the line is not in Cortie's table.

Wave-lengths.	Probable Origins.	Number of Observations.	Relative Mean Widening.	Intensity in Sun.	Remarks.
6494.00	Ca	4	6	6	
95.21	Fe	2	6	8	
99.88	Ca	2	5	4	
6504.42	...	2	5	0	A wv *
09.08	...	6	6	00	A wv * (08.83?)
44.14	...	2	4	2	A wv
46.48	Ti, Fe	1	6	6	
54.47	Ti	4	6	0	*
56.31	Ti	1	6	1	*
63.05	H	53	...	40	See Note (4)

Notes on some of the Lines.

(1) 6090.43. In his preliminary table Rowland assigned this line to Fe, while in the "corrections" it was amended to Ti, V. My photographs show a strong line of V apparently coincident with 6090.43, while titanium shows a line of moderate intensity which appears to agree with 6091.40. Mitchell (*Astrophys. Journ.* vol. xix. p. 358) assigns the former to Fe, V?, and the latter to Ti?.

(2) 6210.90 and 6306.02. These lines were always widened, and are very probably due to scandium. According to Thalén, the two strongest lines of scandium in the red are 6210.0 (intensity 8) and 6304.0 (intensity 10), which would become 6211.0 and 6305.1 when corrected to Rowland's scale. The differences between these and the spot lines are not greater than others which may be found in Thalén's observations as compared with Rowland's. The identification is the more probable, as scandium lines have been noted among the most widened lines in the region more refrangible than D, notably, 5672.05. It is, however, important that the wave-lengths of the red lines of scandium should be redetermined.

(3) 6243.06 and 6243.32. Both lines are weak in the Sun (000 and 1 respectively), and the widening is so great that I have not been able to determine whether both lines are affected or only one of them. Rowland attributes the weaker line to vanadium, while Mitchell assigns both to that element. My photographs show a very strong line in this position, which would correspond better in intensity with the solar line 6243.3 than 6243.0; but, possibly through lack of dispersion, they do not show two lines. It seems reasonable to suppose, however, that the solar line 6243.3 should be attributed to vanadium, and that this is the line most affected in spots.

(4) 6563.05. Special attention was given to the appearance presented by the C line of hydrogen in and near all spots on the disc on twenty-five different days. Treating observations of the same spot made on different days, or of different spots made on the same day, as separate observations, the line was not notably affected in 31 observations; it was displaced or "knotted," but not reversed, on five occasions; reversed and knotted or displaced in 12 observations; and simply reversed in 5 observations. The displacement of the dark line was sometimes towards the red and sometimes towards the violet.

* Indicates that the line is not in Cortie's table.

Comparison with Stonyhurst Observations.

Of the 146 lines contained in the foregoing table of spot lines 109 appear also in Cortie's list, 9 others are probably common to the two, and 28 were not observed by Cortie. Several of the lines not seen by Cortie, or for which slightly different wave-lengths are given, are due to vanadium and titanium, and in view of the presence of so many other lines due to these elements they might be expected to appear in the spots.

Nearly 200 lines recorded by Cortie do not appear in my table, but the great majority of these were only noted as slightly widened. Only 14 of them had a mean widening of 10 or more on Cortie's scale, and even these were seldom observed; 7 of them in fact were only recorded once, 3 of them twice, 1 three times, 1 four times, and 2 six times.

It is probable that the greater number of lines in Cortie's table is to be chiefly accounted for by the fact that his observations include all phases of the sun-spot cycle, whereas mine only cover a short period preceding a maximum. Many of the additional lines are due to iron and nickel, and Cortie states that his observations "confirm the fact that the iron lines, while not displacing other faint lines, are more affected in minimum than in maximum spots."

There is a difference of another kind between the two sets of observations: namely, that in spite of the much smaller total number of my observations, some of the lines were more frequently recorded than at Stonyhurst. The majority of such lines are due to vanadium, 6090·4, for example, being only once recorded by Cortie, as against 15 times by myself. The chromium line 6330·3 is another notable example, having been only twice recorded by Cortie, while I have never missed it when this part of the spectrum was examined in sufficient detail.

These differences, as well as some of those previously mentioned, might conceivably be explained by supposing that the lines of vanadium, titanium, and chromium were generally more strongly developed during the period covered by my observations than in the Stonyhurst period. It is more probable, however, that they are to be accounted for by the difficulty of making complete records of the affected lines when a large region of the spectrum is undertaken.

The agreement between the two series of observations, so far as they are comparable, may be considered very satisfactory on the whole.

Interpretation of Widened Lines.

In attempting to interpret the sun-spot spectrum there is no obvious reason why we should depart in the first instance from the ordinary methods of spectrum analysis. Many of the

characteristic lines are undoubtedly due to familiar elements, and, given perfect observations, it should not be difficult to determine the conditions under which the corresponding vapours exist in spots unless altogether outside our laboratory experience. If, for instance, the vapours are at a temperature not greatly different from that of the vapours which produce the Fraunhofer lines, it might be expected that the relative intensities of the lines of a given element in spots would be the same as those of the corresponding lines in the arc spectrum of the element ; while if the temperature were very different, the intensities of certain lines would be changed in accordance with experimental results.

From this point of view it would appear that what is needed in the observations is not so much a record of the amount of widening of the various lines as of their actual intensities in the spot spectra. The widening of a line may, in fact, generally be regarded as an intensification such as would be produced by an increase in the thickness and density of the absorbing vapour.

The most convenient scale for representing the intensities of the spot lines is furnished by Rowland's estimates of the Fraunhofer lines, and I have accordingly aimed at reducing my observations so as to indicate the intensities of the widened lines on the solar scale. For this purpose several observations of lines of different intensities were made in which the spot line under observation was compared with neighbouring Fraunhofer lines outside the spot spectrum and the equivalent solar intensity directly obtained. The line 6243'3, for example, reaches an intensity in the spots about equal to that of the iron line 6230'9 in the Fraunhofer spectrum, which is given as intensity 8 in Rowland's table ; the intensity of the spot line may therefore be conveniently considered to be 8. In order to utilise the previous observations the widenings of the lines were also noted on the original arbitrary scale, and from the data thus obtained an interpolation table was made so that the equivalent solar intensities of all the lines which had been previously recorded could be approximately derived. In the absence of evidence to the contrary, lines not noted as widened must be assumed to have at least their normal solar intensities in the spots.

A discussion of the observations on this basis strongly suggests that the vapours producing the special selective absorption in spots do not differ very greatly in temperature from those which produce the Fraunhofer lines : that is, within the limits of error of estimation, the relative intensities of the lines of vanadium, &c., in spots are the same as those of the corresponding lines in the Fraunhofer or arc spectrum. The absolute intensities of the lines are, of course, greater in the spot spectrum ; but to account for this it is unnecessary to go beyond the simple supposition that certain vapours are present in spots in greater quantity, and with greater density, as compared with the general absorbing atmosphere of the Sun.

This interpretation of the spot spectrum receives its greatest

support from the discussion of the spot elements which are only represented in the Sun by comparatively faint lines—namely, vanadium, titanium, and chromium. The nature of the evidence will be sufficiently gathered from the part of the titanium spectrum illustrated in fig. 1. The upper spectrum indicates the relative intensities of the Ti lines in the Sun according to Rowland; the middle spectrum shows my own estimates of the intensities of the lines in the arc spectrum, and the lower one indicates the equivalent solar intensities of the lines in spots, determined partly by direct observation and partly by interpolation as already explained. (The line 6085·47 in the Sun is compounded of iron and titanium, and the solar intensity due to the latter is indeterminate.)

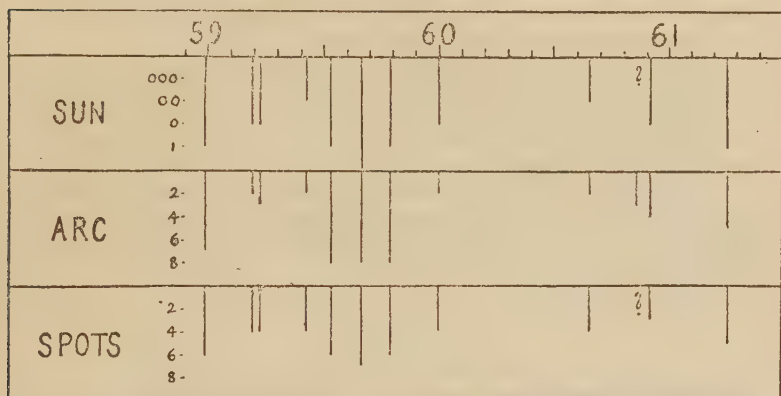


FIG. 1.—The relative intensities of titanium lines in spots compared with the intensities in arc and Sun.

It will be seen that the relative intensities of the lines in spots are in fair agreement with those in the Sun and arc, and the comparison therefore suggests that all the lines of titanium in spots are strengthened in proportion to their intensities in the Sun. Cortie has, in fact, already noticed that the most widened lines of titanium are among the brighter lines due to this element.*

Similar results are obtained for vanadium; and in the case of chromium, which only shows three faint lines in this part of the spectrum, the strongest line in the spots is also the strongest in the arc spectrum. The observations of individual spots lead to the same conclusion, assuming that the occasional omission of lines may be attributed to the difficulty of making complete records.

In the case of spot elements which yield Fraunhofer lines of

* *Mem. R.A.S.* vol. 1. p. 46.

widely different intensities, the weaker lines tend to appear relatively "more widened" than the stronger ones, but it by no means follows that this implies an inversion in the intensities of the lines. It is easy to show that, in photographs at least, a general strengthening of all the lines belonging to an element produces a more obvious effect on the weaker lines than on the strong ones, though all may be intensified in the same ratio. It is therefore probable that the widening of strong solar lines will generally be under-estimated in comparison with that of the weaker ones, or even passed without record. In my own observations, for example, I have never included the strong calcium line $6162\cdot39$ (intensity 15 in Sun) among the affected lines, though many other lines due to the same element were noted; nevertheless the fact that it has been very frequently recorded by Cortie suggests that it was really widened at the time of my observations but was overlooked.

However, if intensities and not "widening" be considered, the results for calcium and sodium are in accordance with those deduced from the behaviour of the lines of vanadium, titanium, and chromium. The strongest lines in the spots are also the strongest in the Sun and arc.

The observations of the lines attributed to iron, nickel, cobalt, and manganese are inadequate for useful discussion, but it is probable that lines of these substances were not much affected during the period of observation.

Telluric Lines.

Several of the widened lines apparently coincide with some of the telluric lines of the Fraunhofer spectrum, a few of them being due to oxygen and a greater number to water vapour.

With regard to the coincidences with lines of water vapour Father Cortie points out that the widening may be really due to faint solar lines in their immediate vicinity, or that the appearance of widening may be fictitious. That the latter is sometimes the case is suggested by the fact that many of the lines in my own table are absent from Cortie's, and *vice versa*. The first suggestion doubtless also applies to some of the lines, for of the lines common to the two series the two most prominent ones at $5938\cdot27$ and $6274\cdot87$ are probably due to titanium and vanadium respectively.

As to the oxygen lines, Father Cortie considers that "the widening of some oxygen lines in sun-spots, particularly in the *a* band (beginning at $6278\cdot30$), seems to be a real phenomenon," but much additional evidence is necessary to establish this view. Only four such lines appear in my table, and the one by far the most frequently observed, both by Cortie and myself ($6306\cdot02$), is in all probability due to scandium (see note 2 to table). It must be remembered also that the lines of oxygen are structurally

related,* and it is, therefore, very improbable that only isolated components would be widened if cool oxygen were really present.

Further investigation of metallic spectra in the red end will doubtless throw additional light on this question.

The Spot Bands.

Four positions are mentioned by Cortie as marked by the occurrence of "spot bands," namely—

6150.36	Band at this position once.
6306.02	A band near here.
6380.96	Spot-band position.
6388.63	" "

In the case of the first two I have not recorded anything different in appearance from the ordinary widened lines, the first being due to vanadium and the second probably to scandium.

I have, however, recorded the remaining two as bands on five occasions, and my most satisfactory estimates of their wavelength are 6381.6 and 6389.0. They seemed to present the appearance of ill-developed flutings, degrading towards the violet, or of narrow bands sharper on the red side, but not extending more than one or two tenth metres. A somewhat different description is given by Mitchell,† who observed on 1904 April 15 that the region 6380 to 6400 was resolved into seventeen groups of fine lines similar in appearance to G under low dispersion.

In four of the spots showing these bands it was noted that the C line was neither reversed nor distorted, while in the fifth (1904 October 29) C was brilliantly reversed just outside the spot, but not in the umbra, where the bands were observed. All the spots in which the bands were seen were comparatively large and the widened lines strongly marked.

An inquiry into the probable origin of the bands has so far been fruitless. They seem to have nothing in common with the titanium flutings which are so characteristic of the Antarian, or third type, stars. Neither are any bands in corresponding positions found on my photographs of the banded spectra of vanadium, chromium, manganese, or iron.

Besides the bands in the red, I have also several times observed those in the region more refrangible than *b*, some of which were first recorded by Maunder, and have recently been photographed by Hale.‡ So far as they go, my estimates of the positions of the bands are in good agreement with those of Hale, as will be seen from the following comparison :—

* Lester, *Astrophys. Journ.* vol. xx. p. 81, among others.

† *Astrophys. Journ.* vol. xix. p. 359.

‡ *Ibid.* vol. xvi. p. 220.

Hale.	Fowler.	Remarks.
5163.72 <i>nb</i>	...	...
5163.66 <i>d</i>	5163.0 <i>d</i>	Maunder 5163.3
5160.15 <i>d</i>	{ 5160.2 5159.8 }	„ 5160.3
5157.86 <i>n</i>	...	...
5156.80 <i>t</i>	{ 5157.2 5156.8 }	Maunder 5157.2
5150.03 <i>d</i>	{ 5150.4 5149.9 }	...
5147.72 <i>d</i> ?	5147.65 <i>n</i>	Ti line
...	5145.64 <i>n</i>	„ „
5144.03 <i>d</i>	{ 5143.9 5143.5 }	...

n = narrow, *b* = band, *d* = double, *t* = triple.

No origin for these bands has been traced, but their occurrence in pairs rather suggests a common origin.

In view of the possible relation of the sun-spot bands to the banded spectra of the fourth-type stars, careful observations were made in the position of the head of the carbon fluting 5165.42, but no evidence of the presence of this fluting in spots was obtained.

All the spot bands thus remain unidentified, and they appear to have no connection with the bands which are characteristic of stars of the third and fourth types.

*Note on the Determination of the Longitude Paris-Greenwich in the Year 1902.**(Communicated by the Astronomer-Royal.)*

In view of the discordance between previous determinations of the longitude of Paris-Greenwich, the International Geodetic Conference in 1898 expressed the view that a redetermination was desirable. It was arranged by M. Lœwy and the Astronomer-Royal that the redetermination should be undertaken in concert by the two observatories of Paris and Greenwich. Owing to the determination of the longitude Greenwich-Killorglin in 1898 and the eclipse expeditions of 1900 and 1901 it was not found practicable to commence this work till 1902.

The programme arranged in conjunction with M. Lœwy provided for the determination being made in two stages—in the spring and autumn of 1902. The instruments used were the four portable reversible transits belonging to the Royal Observatory which had been used in previous longitude determinations from 1888 to 1898. Each observer kept the same instrument throughout, taking it with him from Paris to Greenwich, or Greenwich to Paris. The instruments were distributed thus :—

Observer...	M. Bigourdan	M. Lancelin	Mr. Dyson	Mr. Hollis
Transit instrument }	E	D	B	C

The observing stations at Greenwich were the Transit Pavilion and an adjacent wooden observing hut in the front court; the observing stations at Paris were in the grounds to the south of the Observatory and a little to the east of Cassini's meridian.

The French observers originally selected by M. Lœwy were M. Renan and M. Bigourdan, but owing to the illness of M. Renan he was replaced by M. Lancelin. The English observers were Mr. Dyson and Mr. Hollis.

Independent determinations of longitude were thus made simultaneously by the French and English observers, using adjacent stations and similar instruments. A double interchange of observers and instruments was made both in the spring and autumn. In the autumn the stands were also changed with the instruments. The programme of observations required clock-errors to be determined simultaneously at Greenwich and Paris on three, six, and three full nights respectively in the three parts of each determination. The following table gives details of the number of nights of observation of the English observers :—

Date.	Observer at			Number of Nights on which Clock-error was Determined			Weight.
	Paris.	Greenwich.	Signals Exchanged.	Paris.	Greenwich.	Simultaneously.	
1902.							
I. Mar. 17-30	Dyson	Hollis	13 nights	5	9	4	7
II. Apr. 6-24	Hollis	Dyson	19 „	10	11	6	12
III. Apr. 23-May 3	Dyson	Hollis	7 „	3	5	3	6
I. Sept. 21-26	Hollis	Dyson	6 „	4	6	4	7
II. Sept. 29-Oct. 22	Dyson	Hollis	24 „	11	13	8	14
III. Oct. 25-Nov. 4	Hollis	Dyson	11 „	8	8	5	8

The complete programme for a full night's observations was as follows :—

- I. Observation of mark and nadir.
- II. *Micrometer E.** : Eight time stars, one or more polar stars, two or three observations of level.
- III. *Micrometer W.* : Eight time stars, one or more polar stars, two or three observations of level.
- IV. Observation of nadir.
- V. *Micrometer W.* : Eight time stars, one or more polar stars, two or three observations of level.
- VI. *Micrometer E.* : Eight time stars, one or more polar stars, two or three observations of level.
- VII. Observation of nadir and mark.

Two interchanges of telegraphic signals were made in the course of the evening, signals being sent in each case from Greenwich to Paris, and Paris to Greenwich, and registered on the chronographs both at the sending and receiving stations.

The Collimation Error.—This error was eliminated by the reversal of the instrument. Its constancy, however, as determined in four different ways, viz. directly by reversal on the mark and on the nadir, and indirectly by the agreement of the polar stars observed micrometer East and micrometer West and of the time stars observed micrometer East and micrometer West, gave the most satisfactory proof of the stability of the instrument and fixity of the object glass.

The Level Error was determined both by nadir observations and by the striding level, using the micrometer screw of the level so as to eliminate as far as possible any variation in scale value. As results for the deduced longitude as derived from nadirs and levels separately differ by only 3.013 for the spring determination and 3.002 for that in the autumn, it is clear that no appreciable error in the longitude is attributable to this cause.

The Azimuth Error.—The value of n adopted for each night

* The two positions of the instrument are denoted "Micrometer E." and "Micrometer W."

usually depended on the mean of two or three observations with the micrometer East, and an equal number with the micrometer West. The effect on the clock error is $n(\tan \delta - \tan \lambda)$, where δ is the mean declination of the time stars and λ the latitude. Stars of high declination (but always south of the zenith) were used, so that the above factor is small. The accordance of the separate determinations of n shows that no appreciable error, systematic or accidental, can be attributed to this cause.

Clock-error.—The clocks used were the standard sidereal clocks of the two observatories, Greenwich and Paris. The rates were very uniform. The catalogue of stars used for determination of clock-error was prepared by M. Lœwy, the right ascensions being reduced to Newcomb's fundamental system. At the conclusion of the longitude determination corrections were deduced to the adopted positions of the stars from the combined observations of the French and English observers. These corrections were applied, but did not alter the resulting longitude by $^s.001$. Incidentally the deduction of these corrections showed that the probable accidental error of the observation of a star transit was $\pm ^s.033$.

The Telegraphic Signals.—At each station the signals sent and received were recorded by means of the same relay and local circuit as the observations of transits. By the insertion of suitable resistance in a parallel circuit the current actuating the relay was made the same when recording the observations of transits and of signals received or sent. The time of transmission in one direction was found to be $0^s.021$ from the mean of the spring determinations and $0^s.022$ in the autumn. The differences from these means on individual nights were usually only a few thousandths of a second.

The following table exhibits the separate determinations of the difference of longitude between Cassini's meridian at Paris and that of the transit circle at Greenwich. The difference of the personal equation $D-H$ was found to be $-0^s.0421$ in the spring and $-0^s.0425$ in the autumn. The value $-^s.042$ has been adopted throughout and applied to derive column 3 from column 2.

Difference of Longitude between Paris and Greenwich.—Spring Determination.

		1902.	Determined Difference of Longitude.		Corrected for Personal Equation.		Weight.	Discord- ance from Mean 9 ^m 20 ^s ·977.	Discordance from General Mean 9 ^m 20 ^s ·932.	
			m	s	m	s		s	9 ^m 20 ^s ·932.	
Dyson at Paris Hollis at Greenwich	{	Mar. 19	9	21·060	9	21·018	2	+0·041	+0·086	
		23		·108		·066	1	+0·089	+0·134	
		25		·054		·012	2	+0·035	+0·080	
		28		·096		·054	2	+0·077	+0·122	
Hollis at Paris Dyson at Greenwich	{	Apr. 13	9	20·979	9	21·021	2	+0·044	+0·089	
		17		·966		·008	2	+0·031	+0·076	
		18		·901		20·943	2	−0·034	+0·011	
		20		·940		·982	2	+0·005	+0·050	
		23		·922		·964	2	−0·013	+0·032	
		24		·899		·941	2	−0·036	+0·009	
Dyson at Paris Hollis at Greenwich	{	Apr. 28	9	20·901	9	20·859	2	−0·118	−0·073	
		May 1		·979		·937	2	−0·040	+0·005	
		3		·977		·935	2	−0·042	+0·003	
Mean discordance for Weight 2							...	...	±0·047	±0·059

Autumn Determination

		1902.	Determined Difference of Longitude.		Corrected for Personal Equation.		Weight.	Discord- ance from Mean 9 ^m 20 ^s ·910.	Discordance from General Mean 9 ^m 20 ^s ·932.
			m	s	m	s		s	
Hollis at Paris Dyson at Greenwich	{	Sept. 21	9	20·868	9	20·910	1	0·000	−0·022
		22		·808		·850	2	−0·060	−0·082
		24		·838		·880	2	−0·030	−0·052
		26		·846		·888	2	−0·022	−0·044
Dyson at Paris Hollis at Greenwich	{	Oct. 8	9	20·926	9	20·884	1	−0·026	−0·048
		9		·969		·927	1	+0·017	−0·005
		10		·935		·893	2	−0·017	−0·039
		14		·967		·925	2	+0·015	−0·007
		16		·954		·912	2	+0·002	−0·020
		17		·949		·907	2	−0·003	−0·025
		21		·950		·908	2	−0·002	−0·024
		22		·965		·923	2	+0·013	−0·009
Hollis at Paris Dyson at Greenwich	{	Oct. 27	9	20·919	9	20·961	1	+0·051	+0·029
		29		·874		·916	2	+0·006	−0·016
		Nov. 2		·874		·916	2	+0·006	−0·016
		3		·873		·915	1	+0·005	−0·017
		4		·936		·978	2	+0·068	+0·046
Mean discordance for Weight 2				...	...		±0·019	±0·028	

If the determinations in the spring and autumn are considered separately we find—

(Spring) Long. = $9^{\text{m}} 20^{\text{s}}.977 \pm .0110$.	Prob. error of full night = $\pm .039$
(Autumn) Long. = $9^{\text{m}} 20^{\text{s}}.910 \pm .0042$.	Prob. error of full night = $\pm .016$

The smaller probable error in the second series entitles it to greater weight, but the difference between the two determinations being so much larger than the probable errors would indicate, the two results were not combined with the relative weights given above. If the simple mean is taken $9^{\text{m}} 20^{\text{s}}.943$ and the discordances formed from it, the probable error of a full night (weight 2) in the spring determination is found to be $\pm .051$, and in the autumn determination $\pm .037$. The autumn determination has therefore been given a double weight as compared with that in the spring, and the resulting value of the longitude is

$$9^{\text{m}} 20^{\text{s}}.932 \pm .0060.$$

The discordances from this value are given in the last column.

The systematic difference between Parts I. and III. in the autumn determination is much smaller than in the spring. This is an additional reason for giving a double weight to the latter determination.

It does not seem possible that the systematic discordances shown in the separate parts of the two determinations can be attributed to instrumental errors. Variation of the personal equations of the observers is a natural explanation, and in support of this it may be noticed that the largest discordances are in Series I. of each of the determinations, especially of the one made in the spring. The observers' personal equations had possibly not settled down to the values they subsequently acquired. The advantage of the double exchange of observers and of a determination in the spring and the autumn is its effect on the elimination from the result of such systematic errors.

The value found by the French observers, MM. Bigourdan and Lancelin, is given in the *Comptes Rendus*, vol. cxxxix. p. 1014, as $9^{\text{m}} 20^{\text{s}}.974 \pm .008$.

The results for the several determinations of the Paris-Greenwich longitude by the English observers are :—

	Longitude.	Prob. Error.
	$9^{\text{m}} 20^{\text{s}}.85$	$\pm .018$
1888		
1892	$9^{\text{m}} 20^{\text{s}}.79$	$\pm .022$
1902 I.	$9^{\text{m}} 20^{\text{s}}.977$	$\pm .011$
1902 II.	$9^{\text{m}} 20^{\text{s}}.910$	$\pm .004$

On the Temperature of Sun-spots and the Spectrum of an artificial one. By W. E. Wilson, D.Sc., F.R.S.

The most usual theory of the cause of the darkness of sun-spots is that formulated by De la Rue, Stewart and others, viz. that spots are produced by the down-rush of cooler material on to the photosphere ; and the fact most strongly insisted on in support of this theory is that in the spectrum of a spot there are a considerable number of the Fraunhofer lines, both darkened and widened.

The photograph reproduced on Plate 8 of what I have called the "Spectrum of an Artificial Sun-spot," as I believe it will throw a good deal of doubt on the validity of the usual sun-spot theory, and may possibly go towards proving that in sun-spots we have regions of a higher temperature than the surrounding photosphere, and not a cooler one.

The photograph was obtained by the following experiment. An opal globe surrounding an electric arc represented the Sun's photosphere. A small patch of thin paper on the globe, which reduced its light, say, 50 per cent., was my sun-spot. A rectangular glass cell containing nitric oxide fumes was my chromosphere or absorbing layer. A lens was arranged to throw an image of the paper patch on part of the slit of the spectroscope in the same way as a real sun-spot. The photograph of the spectrum shows that where the dark absorption lines cross the band of general absorption due to the paper patch a good many of the lines are both darkened and widened. I find that all lines which have nebulous edges are widened, and lines that have sharp edges, or the sharp edges of bands are not affected. In fact, in this photograph we have an almost similar effect to that of a real sun-spot without having to call in anything like a cool down-rush, but merely by reducing the radiation from the background.

Now what are the causes which give us a dark-line absorption spectrum ? In the first place the absorbing gas must be at a lower temperature, but also it must be thick enough to be opaque to those particular wave-lengths which give the positions to its dark lines. As we increase the thickness of the absorbing gas we first get the darkest lines, and then more and more of the lines appear, until ultimately the entire continuous spectrum would be blotted out. This shows that for certain thicknesses of the absorbing gas it is quite transparent to some wave-lengths, but to which ultimately it becomes opaque ; and this takes place by no alteration in the temperature. If we have a feeble source of light giving the continuous spectrum, the absorbing layer becomes opaque with a less depth than when the source is of greater brilliancy.

The absorption spectrum of water vapour in the Earth's

atmosphere is a good example of the effect of increasing thickness on the number of lines seen.

If a sun-spot is a region where the temperature is so high that the solid particles which form the photospheric clouds are turned into a gas, it—being then a bad radiator—would appear comparatively dark. It is of importance to remember that a sun-spot is only dark when compared with the dazzling brilliancy of the photospheric clouds, and is really about as bright as the limelight.

It has been argued that if a spot was a really gaseous region, and that it was deep enough to be opaque, it would radiate as a solid; but I think the suggestion made by the late Professor G. Fitzgerald gets over this difficulty. He said that in the Sun there must be in such a gaseous layer enormous convection currents, which would scatter a lot of the light coming from the lower layers, and in fact that it would never reach the surface, so that the general effect would be the radiation from a layer of gas not deep enough to behave as a solid, so that the spot would appear dark.

There seems to be a good deal of evidence that the Sun's photosphere is merely a rather thin cloud stratum the solid particles of which are carbon. In the first place we do not know any other substance that would remain in the solid form at the temperature of, say, $7000^{\circ}\text{C}.$; and from some experiments made by Fitzgerald and myself on the effect of high pressure in the gas surrounding an electric arc we came to the conclusion that carbon at the temperature of $3500^{\circ}\text{C}.$ in the arc is not nearly at its boiling-point, as was generally supposed.

Another point which greatly strengthens the probability of the photosphere being carbon is the observation made lately by Hale. Using the large solar image of the Yerkes refractor, and placing the slit of the spectroscope tangentially as close as possible to the photosphere, he succeeded in seeing as bright lines both the yellow and green bands of the fluted spectrum of carbon.

This observation is, I think, of first importance in showing that in the photosphere we have carbon.*

That there should be a large quantity of carbon in the Sun also seems probable from the wide determination of it on the Earth. Its atomic weight would also give it a position with other gases that we know occupy this level.

Now is it not conceivable that at a certain distance below the photosphere the temperature may be so high that even carbon is volatilised, and that in a sun-spot we have a local upheaval of this high temperature which volatilises the solid particles of carbon and enables us to see below the photospheric clouds? A spot being then a gaseous layer, it would part with

* A fluted spectrum was generally considered a very low-temperature one, but Hale's observation shows that carbon gives such a spectrum at a temperature of at least $7000^{\circ}\text{C}.$, or twice the temperature of the arc.

its heat much slower than the photosphere, and thus have its life prolonged.

It has been urged that, even if the solar temperature was raised enough to volatilise the photospheric clouds, they would merely reform again at a greater altitude where the temperature was lower ; but it is easy to see that the carbon vapour could not reach a sufficient altitude to reform into a cloud layer. If this was so we would have cloud layers formed at suitable altitudes of iron, titanium, magnesium, and many others ; and the fact that these do not exist is, I think, a proof that carbon also, with a suitable rise of temperature, would be unable to remain in the solid form.

It therefore seems that brilliancy *per se* is no criterion as to the temperature of a star, and if the solar temperature was raised a certain amount his brilliancy would fall probably 50 per cent., his spectrum would show a considerable number of the lines of calcium, magnesium, and others as bright lines, while the lines of substances like titanium and vanadium, which lie below the photosphere, and the bands of carbon would be darkened and widened. In fact, the solar spectrum would become almost identical to that of IV. type stars, which should therefore be classed as hotter and not cooler than the Sun.

Professor Hale has pointed out lately that the spectrum of these stars is almost identical with that of a sun-spot.

Mr. W. M. Mitchell has recorded in the *Astrophysical Journal*, 1904 June, that out of 270 lines affected in some spots which he observed during March and April sixty-eight were reversed. Now why should we see lines reversed in spots ? Certainly not by the supposed down-rush of cool material, but possibly by want of radiation from the layer giving the continuous spectrum, or by a very high temperature of the gas giving the line spectrum. By the first cause we can see the hydrogen lines bright on the Sun's limb, when we then have as a background the feeble spectrum of the lower corona. If the radiation from the umbra of a spot was as feeble as the corona close to the Sun's limb, then we should see the hydrogen lines as bright as we do in a prominence.

In the Sun there seems to be two distinct layers which give us a continuous spectrum : first, the photospheric clouds ; and secondly, at some depth below them, a layer of gases of sufficient thickness and pressure to be opaque. Between these two layers lie the vapours of titanium, vanadium, and other elements of suitable atomic weight. Below this gaseous layer probably lie the elements like uranium with high atomic weight ; and unless they are occasionally pushed up by convection currents, their spectrum will never be seen.

As we approach the photosphere probably the vapour of all or most of the elements that lie beneath it and above the gaseous layer get mixed up and carried above it, and thus give us the ordinary Fraunhofer spectrum. It is this mixed layer of hot

gases that gives us the flash, and it evidently cannot rise much above the photosphere. In a sun-spot then the causes which give rise to the widening and darkening of some of the lines is partially due to the want of brilliancy of the gaseous layer below, and also to the greater depth of the absorbing vapours of the elements like titanium, &c., whose atomic weight gives them a place between the photosphere and the gaseous layer. The photospheric clouds seem to be not thick enough to be quite opaque, and a certain amount of light breaks through by the pores from the gaseous layer below. This helps to make the Fraunhofer lines darker than they would be if the photosphere was quite opaque to light coming from below. If the photosphere was of sufficient depth to be quite opaque, and also if the vapours of the elements lying below it could not reach the surface by convection currents, then only elements like hydrogen with a lighter atomic weight than twelve of carbon could lie above it, and we would get a spectrum like that of *Sirius*. As there would be only a few dark lines of hydrogen to absorb the light of the continuous spectrum the brilliancy of the star would be greatly enhanced principally in the violet end. This would not suggest, as Sir N. Lockyer urges, a higher temperature than the Sun, but one slightly cooler and not so near the critical temperature at which carbon can exist in the solid form.

Langenbach has shown that in a bright line spectrum as the temperature rises the maximum intensity shifts towards the violet end. Campbell finds that the Wolf-Rayet star D.M. + 30° 3639 has an extensive hydrogen atmosphere, and that $H\alpha$ is very faint, while $H\gamma$ is brighter, and $H\beta$ very much brighter. This seems an interesting point in favour of the very high temperature of this class of stars.

As a summary of this paper I maintain that in the Sun, and also in stars of the same or lower temperature, we have two distinct layers which give us a continuous spectrum. First, a gaseous one of high temperature and pressure; and secondly, a layer of carbon clouds which are a far greater radiator than the first gaseous layer. In sun-spots where the temperature is locally high enough to volatilise the carbon clouds we get radiation from the gaseous layer alone and the absorption spectrum much intensified by those vapours which lie between these two layers. In stars like *Sirius* we get the spectrum due to a much greater depth of carbon clouds than we have in the Sun, and outside of which we have principally a great atmosphere of hydrogen. In IV. type and Wolf-Rayet stars we have bodies at too high a temperature for carbon clouds to form, and we have the continuous spectrum only from the gaseous layer, which is darkened also by the powerful absorption spectrum of carbon, titanium, and other elements with which the stars' atmosphere is charged.

The Spiral Nebula H I. 153 Ceti. By W. S. Franks.

In revising the last working list at Starfield several objects were included that had previously been omitted on account of their great southern declination. The nebula H I. 153 was one of these; so, taking advantage of an exceptionally clear sky on the evening of 1905 January 1 (when *Sirius* could be seen close to the horizon), I thought it was a favourable opportunity for attacking an object so long neglected. Although one of H's Class I., it should, from its actinic effect, be relegated to Class II., for it is very faint on the original negative, and necessitated a repeated copy to bring it up to printing density (Plate 8).

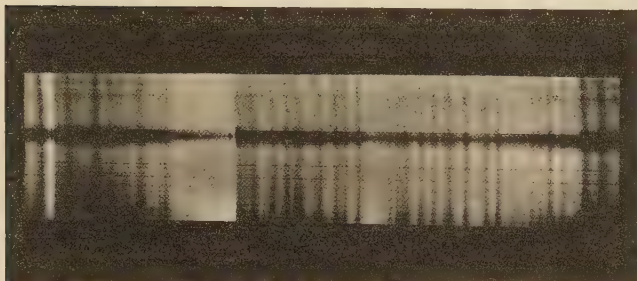
Coming now to details, the object is N.G.C. 908, G.C. 536, H I. 153, R.A. $2^h 18^m 28^s$, Decl. $-21^\circ 41' 3''$ (1900). It is described in the N.G.C. as "considerably bright, very large, extended." The photograph was obtained on 1905 January 1, with 90^m exposure in the 20-inch reflector, between $2^h 7^m$ and $3^h 37^m$ local sidereal time; very clear and severe frost. The photograph at once shows it to be a rather interesting left-handed spiral, viewed somewhat obliquely. It is about $4'$ long by about $1\frac{1}{2}'$ broad, and its major axis lies at about pos. ang. $80^\circ \pm$. There is a central condensation, and a detached small star about $3'$ following the nucleus; also a brighter star about $10'$ following. A peculiar feature is the bifurcation of the following arm of spiral. I do not remember a parallel case in an object of this kind. The already long list of spiral nebulae gains yet another accession by the inclusion of this object. The accompanying illustration will assist the foregoing description. The scale is approximately $12''$ of arc to 1 millimetre.

Starfield Observatory, Crowborough Beacon.

Further Note on the Origin of Magnitude-Equation in Photographic Measures. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. In *Monthly Notices*, vol. lxxv. p. 54, I drew attention to the fact that a faulty objective may produce images showing a magnitude-equation which has not hitherto, so far as I know, been considered; and further reflection and discussion have not altered my opinion that such a possibility must always be taken into account in discussing photographic measures for very small quantities, as in parallax work. But a recent conversation with the Astronomer-Royal suggests that the precise form of fault in the objective was probably not correctly assigned; and it may

(DR. W. E. WILSON'S PAPER.)



SPECTRUM OF ARTIFICIAL SUN-SPOT.

(MR. W. S. FRANKS' PAPER.)



SPIRAL NEBULA $M 1.153$ CETI

Photographed by W. S. FRANKS.

be useful to others to know beforehand what to look for if there is so conspicuous an effect as to suggest mechanical correction.

2. The remark of the Astronomer-Royal is briefly to the effect that an error of *centring* of one lens, *i.e.* a lateral displacement of one lens relatively to the other, is not so likely to produce serious effects as a relative *tilt*. This he found by personal experience when adjusting the 28-inch objective at the Royal Observatory. There was originally a defect which gave unsymmetrical out-of-focus images, and he in the first instance attributed it to defective centring as above, but found on trial that one lens could be moved laterally with reference to the other by quantities of the order of a quarter of an inch without serious change of the images. He then tried relative tilt, and found that a very small correction of that kind gave the required results. Hence it seems more likely that we should look for defects in this direction, especially if the lenses of an objective are sensibly separated.

3. The defect in the Algiers objective, which gave rise to the well-marked magnitude-equation, was apparently of this kind, though arising in rather a different way. I gather from M. Trépiéd that a representative from M. Gautier found the lenses of the objective pressed too tightly together. Such pressure would be very unlikely to be quite symmetrical, and the effect might be regarded as a form of tilt.

4. With heavy objectives it seems desirable to be on the watch for an error of this kind depending on the pressure produced by the weight of the objective itself. Such errors would, it is to be hoped, be very small; but they might come within the scope of very refined investigations; and they might unfortunately become serious from the enormous labour required to investigate them. They would, in each individual plate, be associated with "driving error," or correction-of-refraction error which might be of the same kind; and could only be determined from the mean of a large number of plates.

On the Possible Effects of Radiation on the Motion of Comets, with special reference to Encke's Comet. By H. C. Plummer, M.A.

1. It may be said with confidence that the theory of radiation, which occupies a prominent place in modern physics, possesses an importance in astronomy which is only beginning to be appreciated. It has been suggested that the repulsive force manifested in the tails of comets finds its simplest explanation in this theory, and quite recently Arrhenius has appealed to the properties of radiation in a discussion of the physical nature of the solar corona. To Professor Poynting we are indebted for a memoir on

"Radiation in the Solar System" (published in the *Philosophical Transactions* and reprinted in the *Memoirs* of this Society), which supplies a lucid exposition of those details of the subject which mainly concern the astronomer. The discussion of the effects of radiation pressure, which is contained in the second part of his paper, suggests many points of great astronomical interest, and may well lead us to reconsider some of the conclusions hitherto held. Naturally it is in the sphere of meteoric astronomy that the influence is mainly to be expected. To take an example, two spherical meteors of density 6.2 and radius 1 cm., possessing the temperature which they acquire at the same distance from the Sun as the Earth, neither attract nor repel one another, the gravitational stress being balanced by the radiation pressure. Facts such as this may well lead to a revision of the views at present held as to the stability of meteor swarms, the subject of a series of well-known researches by Schiaparelli, Charlier, L. Picart and Callandreau. As Poynting suggests, such facts may have a bearing on the theory of Saturn's rings provided the temperature of the planet is sufficiently high. Most significant of all, from the theoretical point of view, is the modification, though not of course the destruction in the physical sense, of the law of action and reaction. It seems just conceivable that the transformation of material momentum into ether-momentum may have important influences on a cosmic scale.

2. If the meteoric hypothesis as to the constitution of comets be accepted, the results which apply to meteors can be extended to comets. The degree of concentration of the meteors must be sufficient to account for the luminosity of the comet and yet not so great but that each meteor may have a practically unobstructed view of the Sun. For the sake of simplicity it is assumed that the meteors are spherical in shape, and that they are perfect absorbers of radiation, though this cannot be the case. If r is the distance from the Sun, the gravitation pull may be denoted by μ/r^2 and the radiation pressure by μ'/r^2 . Using the formula and constants given by Poynting (p. 540*), we have

$$\frac{\mu'}{\mu} = \frac{7.4 \times 10^{-5}}{a\rho}$$

where a is the radius of the meteoric particle and ρ its density, the C.G.S. system of units being adopted. Now if a is the mean distance and n the mean motion of the particle, the expression of Kepler's third law is

$$n^2 a^3 = \mu$$

where μ is a constant depending only on the mass of the Sun and on the system of fundamental units employed. But when allow-

* The reference here and in § 5 is to *Phil. Trans.* vol. 202 A.

ance is made for the effect of radiation pressure the law is modified so that its expression becomes

$$n^2 a^3 = \mu - \mu'$$

The change implies a virtual reduction in the Sun's mass. If μ'/μ is one-millionth and ρ is taken to be about 5.5, the corresponding value of a is about 13 cm. If the size of the constituent meteors reaches or exceeds this limit it is scarcely likely, with the present degree of accuracy in comet observations, that the departure from Kepler's law can be detected. But the meteors may very easily be considerably less than 10 inches in diameter, and in that case it should be possible to find evidence of the action of radiation pressure.

3. There is no difficulty in finding discrepancies between theory and observation in the orbital motions of comets, and the important question which confronts us is this : How far can such discrepancies be explained by the fact that an incorrect value may have been assumed of the effective mass of the Sun ? The intention here is simply to suggest this question, not to give any definite answer to it. The case which has received the greatest amount of study is that of Encke's comet. The elements calculated for one apparition and corrected for the perturbations caused by known sources of disturbance during the interval in which the comet is beyond the range of observation are not satisfied by the observations made at the following apparition. To meet the difficulty it has been usual, from the time of Encke, to assign an empirical variation to the mean motion. This is explained by the hypothesis of a resisting medium for which there is no confirmatory evidence, and which seems to have been introduced simply *ad hoc*. Moreover, since the variation of the mean motion has not remained constant in all revolutions of the comet, it has been necessary to attribute properties to the medium which seem rather artificial. However this may be, the relation connecting the mean distance and the mean motion (thus varied) with what may be called the planetary mass of the Sun has presumably been preserved. If this assumption has in fact been made it is an important question how far the discrepancies between theory and observation can be explained by making allowance for the pressure due to solar radiation. Possibly a negative answer is implicitly contained in those works to which Dr. Backlund has devoted so much skill and labour. But if not, an interesting though laborious research is suggested, which has not yet been undertaken. The point here insisted on is that in the case of all comets it is unwarrantable to assume that the mean motion and the mean distance are related and not independent elements.

4. It has been supposed in what precedes that the size of the particles of which a comet is composed is uniform and permanent. This may not be the case. Collisions may cause them to coalesce,

and so to increase progressively in size. The radiation pressure will thus be diminished and an acceleration of the mean motion may result from this cause. At the same time there will be a corresponding diminution in the brightness of the comet. It is easily seen, in fact, that if m spherical particles coalesce into n meteors both the radiation pressure, expressed in terms of the corresponding gravitative pull, and the brightness of the comet are changed in the same ratio—namely, the cube root of $m : n$. Other causes may be imagined for an alteration, either continuous or discontinuous, in the size of the meteors. Hence, even if their size is supposed originally uniform, it will not necessarily remain so. The pressure due to solar radiation may thus through its selective action become a powerful factor in the disintegration of comets. Is it possible that we have here an explanation of the anomalous behaviour of Biela's comet? If from any cause or combination of causes the constituent particles became separated into two distinct sizes, the division of the nucleus follows as a natural consequence. The idea that the dimensions of the meteoric constituents were wanting in stability is supported by the remarkable variations in the relative brightness of the two nuclei and by the ultimate disappearance of the comet, which may have been due either to its transformation into a meteor swarm in the ordinary sense, and its consequent loss of brilliancy, or to its final disintegration. Another inference is suggested by the possibility that the effective attraction of the Sun on a comet is not necessarily constant. In order to disprove the identity of origin of two comets it does not seem sufficient to show that their elements do not satisfy Tisserand's criterion.

5. A further consequence of the radiation theory naturally presents itself as a plausible reason for the acceleration of the mean motion of a comet, and seems to deserve careful examination although the result will be found to be a negative one. This is derived from the application of Doppler's Principle to the radiation pressure against a moving surface. The result found by Poynting (p. 548, corrected by the note, p. 550) is that the force acting against a spherical particle which moves with the velocity v produces a tangential retardation Tr/r^2 , where

$$T = Sb^2/4U^2\rho\alpha,$$

S being the solar constant at the Earth's distance, b , from the Sun, and U the velocity of light. The action is clearly precisely analogous to that of a resisting medium of the type commonly assumed, and will tend to accelerate the mean motion in the same way, although the physical nature of the action is entirely different. There is a corresponding diminution in the distance from the Sun, a numerical example of which may be taken from Poynting's paper: a sphere of radius 1 cm. and density 5.5, starting with the velocity of the Earth and at its distance from the Sun will move inwards $\frac{1}{10000}$ of its distance in about 20,000

years. This may be otherwise expressed by saying that its period of revolution is diminished by 0.2 sec. a year. If the same laws were applicable to a body actually as large as the Earth, the result would be to diminish the length of the year by 1 second in 3×10^9 years. The concrete question suggested by such results is whether the effect can reach an order which will account for the anomalous features which have been observed in the motion of Encke's comet.

6. Since T is constant for all distances and of an order such that its first power alone need be retained, the perturbations of an elliptic orbit, being entirely in the plane of motion, can be calculated very simply. The equations of motion can be written

$$\ddot{r} - r\dot{\theta}^2 = -\frac{\mu - \mu'}{r^2} - \frac{T}{r^2} \dot{r}$$

$$\frac{d}{dt}(r^2\dot{\theta}) = -T \frac{d\theta}{dt}$$

Integrating the latter and then eliminating t from the former, we get

$$r^2\dot{\theta} = T/c - T\theta$$

$$\frac{d^2u}{d\theta^2} + u = \frac{(\mu - \mu')c^2}{T^2(1 - c\theta)^2}$$

where u is the reciprocal of r , and c is an integration constant of the same order of magnitude as T . Hence the last equation may be replaced by the approximate form

$$\frac{d^2u}{d\theta^2} + u = (\mu - \mu') (1 + 2c\theta)c^2/T^2$$

of which the integral is

$$u = (\mu - \mu') [1 + e \cos(\theta - \gamma) + 2c\theta]c^2/T^2$$

Now it is evident, on general grounds, that the departure from a purely elliptic orbit in any one revolution is small. The preceding equation may therefore be considered a valid representation, during a small number of revolutions, of the motion of a particle which would, in the absence of the effect now considered, describe the elliptic orbit

$$u = (\mu - \mu') [1 + e \cos(\theta - \gamma)]c^2/T^2$$

7. For the sake of definiteness and simplicity we may consider the motion at successive returns to the position which corresponds to the perihelion in the undisturbed orbit and put $\gamma = 0$. After k returns the motion is given by

$$u = (\mu - \mu') [1 + e \cos \theta + 2c(2\pi k + \theta)]c^2/T^2$$

where θ increases from the value 0. The osculating orbit at the point $\theta = 0$ is therefore

$$u = (\mu - \mu') [1 + e \cos \theta + 2c(2\pi k + \sin \theta)]c^2/T^2$$

for this is an ellipse with its focus at the Sun and possessing the same circle of curvature as the disturbed orbit; and since the disturbing force has no normal component, equality of curvature implies equality of velocity in this ellipse and in the actual orbit. Writing the equation in a form in which the meaning of the constants is obvious, namely

$$lu = 1 + e' \cos (\theta - \beta)$$

we have

$$l^{-1} = (\mu - \mu') (1 + 4\pi ck)c^2/T^2$$

$$\tan \beta = 2c/e$$

$$e'/l = (\mu - \mu') (e^2 + 4c^2)^{1/2}c^2/T^2$$

These give the increments of the constants at each revolution. The position of the apse-line is constant and the variations of l and e' (of which the accent becomes superfluous) are given by

$$\frac{\delta l}{l} = \frac{\delta e}{e} = -4\pi c$$

Since $l = a(1 - e^2)$, we find

$$\frac{\delta a}{a} = -\frac{1 + e^2}{1 - e^2} \cdot 4\pi c$$

which leads further, since $n^2 a^3 = \text{constant}$, to

$$\frac{\delta n}{n} = \frac{1 + e^2}{1 - e^2} \cdot 6\pi c$$

8. The last result can be verified by a direct calculation of the periodic time P after k revolutions. For

$$\begin{aligned} P &= \int_{2k\pi}^{2(k+1)\pi} \frac{c \cdot d\theta}{u^2 T (1 - c\theta)} \\ &= \frac{T^3}{c^3 (\mu - \mu')^2} \int_{2k\pi}^{2(k+1)\pi} \frac{d\theta}{(1 + e \cos \theta + 2c\theta)^2 (1 - c\theta)} \\ &= \frac{T^3}{c^3 (\mu - \mu')^2} \int_{2k\pi}^{2(k+1)\pi} \left\{ \frac{1}{(1 + e \cos \theta)^2} - \frac{c\theta(3 - e \cos \theta)}{(1 + e \cos \theta)^3} \right\} d\theta \\ &= \frac{2T^3}{c^3 (\mu - \mu')^2} \{I_2 - \pi c(2k + 1)(4I_3 - I_2)\} \end{aligned}$$

where

$$I_5 = \int_0^\pi \frac{d\theta}{(1 + e \cos \theta)^5}$$

Now

$$I_2 = \pi(1 - e^2)^{-\frac{1}{2}}.$$

$$I_3 = \frac{1}{2}\pi(2 + e^2)(1 - e^2)^{-\frac{1}{2}}$$

Hence

$$P = \frac{2\pi T^3}{c^3(\mu - \mu')^2} (1 - e^2)^{-\frac{1}{2}} \left\{ 1 - 3\pi c(2k + 1) \frac{1 + e^2}{1 - e^2} \right\}$$

This is the period in the $(k + 1)$ th revolution, and consequently the variation of the period is given by

$$\frac{\delta P}{P} = -6\pi c \frac{1 + e^2}{1 - e^2}$$

which verifies the result found for the mean motion.

9. Now T/c is twice the areal velocity; hence if q is the perihelion distance

$$T/c = q^2 n (1 + e)^{\frac{1}{2}} (1 - e)^{-\frac{1}{2}}$$

Therefore

$$\delta n = 6\pi T (1 + e^2) (1 + e)^{-\frac{1}{2}} (1 - e)^{\frac{1}{2}} / q^2$$

Using 1.75×10^6 as the value of S and 3×10^{10} as the value of U in the expression given for T (§ 5), we have

$$6\pi T = 9.2 \times 10^{-15} b^2 / \rho a$$

Hence

$$\delta n = 9.2 \times 10^{-15} (1 + e^2) (1 + e)^{-\frac{1}{2}} (1 - e)^{\frac{1}{2}} b^2 / q^2 \rho a$$

or

$$\rho a = 9.2 \times 10^{-15} \frac{(1 + e^2)(1 - e)^{\frac{1}{2}}}{(1 + e)^{\frac{1}{2}}} \cdot \frac{b^2}{q^2} \cdot \frac{1}{\delta n} \dots \dots (1)$$

which is the formula (in C.G.S. units), giving ρa when δn is known. Moreover, by connecting the expressions for δe and δn in terms of c (§ 7), we have

$$\frac{\delta e}{e} = -\frac{2}{3} \cdot \frac{1 - e^2}{1 + e^2} \cdot \frac{\delta n}{n}$$

or putting

$$e = \sin \phi$$

$$\delta \phi = -\frac{1}{3} \cdot \frac{\sin 2\phi}{1 + \sin^2 \phi} \cdot \frac{\delta n}{n} \dots \dots (2)$$

10. To represent the conditions of Encke's comet it is sufficient to take $q = \frac{1}{3}b$, $e = 0.85$, $\phi = 58^\circ$, and $n = 1070''$ per day. In accordance with Backlund's results, derived from the more recent apparitions, we may take $0''.068$ as the increment of the mean daily motion in a revolution. The result of substituting these numbers in (2) is to give $\delta \phi = -2''.3$, while Backlund's corresponding result is $-2''.4$. The agreement between the two

numbers is, however, only apparently remarkable, for its significance is scarcely greater than that of an arithmetical check. As Backlund has pointed out, the relation between the variations δn and $\delta \phi$ which arise from the action of a resisting medium is practically independent of the nature of the medium, and can be assumed to be known. Hence, the action of the disturbing force here considered being formally identical with that of a resisting medium of simple type, it follows that the agreement is in reality nothing more than ought to be expected.

11. By expressing the above value of δn with a second as the unit of time and reducing to circular measure, we obtain

$$\delta n = 1/2.62 \times 10^{11}$$

Hence the formula (1) gives

$$\begin{aligned} \rho a &= 9.2 \times 10^{-15} \times 0.265 \times 9 \times 2.62 \times 10^{11} \\ &= 5.7 \times 10^{-3} \end{aligned}$$

This shows that if we assume the density to be about equal to the mean density of the Earth a particle whose radius is one-hundredth of a millimetre and whose orbit round the Sun is identical with that of Encke's comet will have its mean motion accelerated by the same amount as has actually been determined from observations at the recent apparitions of the comet. But it does not follow from this that the periods of the comet and the particle will be the same. It is necessary to consider the effect of the direct radiation pressure. Now, by using the formula for μ' (§ 2) the above value of ρa gives

$$\mu'/\mu = 0.013$$

so that the radiation pressure amounts to one-eightieth of the force due to solar gravitation. If the effect of this reduction in the solar attraction be attributed to errors in the assumed mean motion and mean distance, the relation between such errors is

$$\frac{2\Delta n}{n} + \frac{3\Delta a}{a} = \frac{\Delta \mu}{\mu} = -0.013$$

On the assumption that the proportional errors in both elements are the same, each would be of the order 1 in 400. This implies an increase of the period by three days, which is scarcely possible! Nor is it easy to believe that the difficulty can be surmounted by the hypothesis of a sufficiently large error in the adopted mean distance. Consequently the difficulty of reconciling the suggested theory with the presumable state of the facts seems to be too great, and unless some serious error has been overlooked the attempt to explain the acceleration in the mean motion of Encke's comet by this particular deduction from

the theory of radiation must apparently be regarded as an interesting but untenable speculation.

12. This, however, is not a conclusion from which there is definitely no escape. We are not so much seeking for a cause of the known peculiarity of the motion of Encke's comet as looking to that peculiarity for evidence of an actual physical process. Hence it seems unwise to dismiss hastily a theory in some ways so attractive. It may be possible, for instance, that the diminution in the solar attraction is neutralised by some other action such as electrostatic attraction. And apart from this possibility there is a consideration of a different kind. We have assumed a meteoric constitution for the comet, and it is difficult to avoid some such theory because the notion that a comet is composed of continuous matter seems impossible. Being widely dispersed the meteors must be practically unaffected by their mutual attractions, and may be considered to describe independent orbits. It is to *each member* of the total aggregate of orbits that the foregoing investigation applies, while the observations and the elements based on them refer to an object resulting indeed from the same aggregate, but connected with the individual members in a vague and undefined manner. Since we cannot observe each distinct meteor and trace its relative path within the swarm it is not necessary to assume that the mean distance which is deduced from the observations, and which has a significance purely statistical and quite possibly misleading, is identical with the mean distance of any particular meteor or the true average of the total number. This distinction, for which the theory of waves presents some analogy in the difference between group-velocity and wave-velocity, may be illustrated by the simplest possible example: if two particles describe similar coplanar orbits differing only in the fact that their axes are inclined to one another at an angle 2ε , the point midway between them also describes a similar orbit, but with a mean distance which bears to the mean distance of each particle the ratio $\cos \varepsilon : 1$. The difficulty of assigning a definite dynamical meaning to the point to which the observations refer is further emphasised by the systematic differences which have been found to exist between observations made with large and with small telescopes. The great obstacle in the way of a satisfactory study of the statistical conditions lies in the difficulty of formulating an appropriate hypothesis regarding the complex motions of the constituents of the swarm.

University Observatory, Oxford:
1905 January 11.

On the Validity of Meteor Radiants deduced from Three Tracks.
By H. W. Chapman, B.Sc., University College, London.

(Communicated by L. N. G. Filon, M.A., D.Sc.)

1. It is not uncommon in tables of radiants of meteors to find radiants deduced from very few observations: four is quite a common number, and three by no means rare. We see, however, that it is quite possible that three or four paths, although quite unconnected, should happen to pass so closely through the same point that they would be taken to indicate a radiant there. It therefore becomes of interest to determine the probability of this happening. Suppose the mean divergence of an observed path from the radiant is ω , then it is clear that if the inradius of the Δ formed by the paths of three meteors $\gtrsim \omega$ the three paths will be considered to pass through the same point and, subject to certain conditions which will be investigated later, to indicate a radiant there.

The problem before us is, therefore, to find the chance that the incircle of the Δ formed by three great circles taken at random on a sphere $\gtrsim \omega$.

2. A uniform distribution of great circles on a sphere corresponds to a uniform distribution of their poles; accordingly we must have the circles so placed that it is possible to find a pole of each forming a Δ of circumradius $> \frac{\pi}{2} - \omega$.

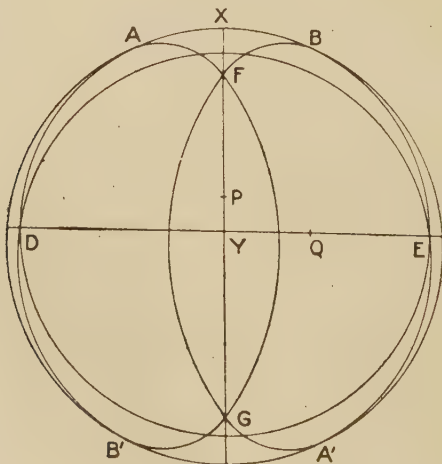


FIG. 1.

Let A (fig. 1) be a pole of one of the circles, and B that pole of the second which is nearer to A.

The chance of AB falling between x and $x+dx$

$$= \frac{2\pi \sin x dx}{2\pi} \sin x dx.$$

Then if $x < 2\omega$ any $\odot$ through A and B', the point antipodal to B has a radius $> \frac{1}{2}\pi - \omega$, and the conditions are certainly satisfied. If $x > 2\omega$, let A' be the point antipodal to A, and through A, B; A, B'; A', B; and A', B' draw circles of radius $\frac{1}{2}\pi - \omega$. We will for the present only consider the four of the eight circles whose major arcs lie on one of the two hemispheres into which the sphere is divided by the plane ABA'B'.

3. We will now find the condition that the circles through A, B; A', B' may cut each other. Let P be the centre of the $\odot$ through AB, and draw PX perp. to AB. Then

$$\cos PX = \cos AP \sec AX = \sin \omega \sec \frac{1}{2}x,$$

and we see that the circles will cut if

$$2 \{PX + (\frac{1}{2}\pi - \omega)\} > \pi,$$

i.e.

$$PX > \omega,$$

i.e.

$$\sin \omega \sec \frac{1}{2}x < \cos \omega,$$

i.e.

$$\cos \frac{1}{2}x > \tan \omega.$$

Since $x < \frac{\pi}{2}$, this is always true if $\omega < \tan^{-1} \frac{1}{\sqrt{2}}$, which is the case, ω being small.

$\therefore$ the circles through A, B; A', B' will cut; let them cut in D, E (fig. 2).

Similarly the circles through A, B'; A', B will cut if

$$\sin \frac{x}{2} > \tan \omega,$$

i.e.

$$x > 2 \sin^{-1} \tan \omega.$$

Now it will be seen that the conditions will not be satisfied if C, the pole of the third circle which lies in the hemisphere considered, fall within all four of the circles through A, B; A, B'; A', B; A', B', but will in every other case.

The conditions will be satisfied if C falls within both of the circles through any one of our four pairs of points; but this gives nothing new, for the second circle through AB is antipodal to A'B'D and so can have no points in common with it, so that our conditions are already satisfied by C being outside A'B'D, and similarly for the other pairs of points.

Now if the circles through A', B; A, B' do not cut, the four

circles will have no common portion and the conditions will be satisfied. Now these circles will not cut unless $x > 2 \sin^{-1} \tan \omega$,

$$\begin{aligned} \therefore \text{the chance in this way} &= \int_0^{2 \sin^{-1} \tan \omega} \sin x \, dx \\ &= 1 - \cos 2(\sin^{-1} \tan \omega) \\ &= 2 \tan^2 \omega \quad \dots \quad \dots \quad \dots \quad (1) \end{aligned}$$

4. Consider now the case when the circles through A, B'; A', B cut in F, G.

Then we have two subcases, according as the points F, G lie both inside or both outside of the area common to ABD, A'B'D. X, P, F, G will clearly lie on the same great $\odot$.

Then F and G will both lie inside the area common to ABD, ABD' if

$$\begin{aligned} FG &< 2 \left\{ \frac{1}{2}(\pi - \omega + PX) - \frac{1}{2}\pi \right\} \\ &< 2(PX - \omega). \end{aligned}$$

To find FG, let Q be the centre of A'B'G and draw QY perp. to FG. Then $\cos \frac{1}{2}FG = \cos QF \sec QY$.

Let QY cut AB A'B' in Z.

$$\begin{aligned} \text{Then} \quad QY &= \frac{1}{2}\pi - QZ \\ &= \frac{1}{2}\pi - \cos^{-1}(\cos QB \sec BZ) \\ &= \frac{1}{2}\pi - \cos^{-1}(\sin \omega \operatorname{cosec} \frac{1}{2}x) \end{aligned}$$

$$\text{and} \quad QF = \frac{1}{2}\pi - \omega;$$

$$\begin{aligned} \therefore \cos \frac{1}{2}FG &= \sin \omega \operatorname{cosec} \cos^{-1}(\sin \omega \operatorname{cosec} \frac{1}{2}x) \\ &= \sin \omega \sin \frac{1}{2}x (\sin^2 \frac{1}{2}x - \sin^2 \omega)^{-\frac{1}{2}}. \end{aligned}$$

No ambiguity exists as to the sign of the root, as $\frac{1}{2}FG < \frac{1}{2}\pi$; $\therefore$ our position becomes

$$\begin{aligned} \cos^{-1} \{ \sin \omega \sin \frac{1}{2}x (\sin^2 \frac{1}{2}x - \sin^2 \omega)^{-\frac{1}{2}} \} &< \cos^{-1}(\sin \omega \sec \frac{1}{2}x) - \omega, \\ \text{i.e.} \quad \sin \omega \sin \frac{1}{2}x (\sin^2 \frac{1}{2}x - \sin^2 \omega)^{-\frac{1}{2}} &> \sin \omega \cos \omega \sec \frac{1}{2}x \\ &\quad + \sin \omega \sec \frac{1}{2}x (\cos^2 \frac{1}{2}x - \sin^2 \omega)^{\frac{1}{2}}. \end{aligned}$$

This on reduction gives

$$\begin{aligned} 4 \sin^4 \frac{1}{2}x - \sin^2 \frac{1}{2}x (8 - 5 \cos^2 \omega) + 4 \sin^2 \omega &> 0, \\ \text{i.e.} \quad \sin^2 \frac{1}{2}x &< \frac{1}{8} \{ 8 - 5 \cos^2 \omega - \cos \omega (25 \cos^2 \omega - 16)^{\frac{1}{2}} \} \\ \text{or} \quad &> \frac{1}{8} \{ 8 - 5 \cos^2 \omega + \cos \omega (25 \cos^2 \omega - 16)^{\frac{1}{2}} \}. \end{aligned}$$

The second of these alternatives is impossible, for $\sin^2 \frac{1}{2}x > \frac{1}{2}$, and if we put

$$\frac{1}{8} \{ 8 - 5 \cos^2 \omega + \cos \omega (25 \cos^2 \omega - 16)^{\frac{1}{2}} \} = \frac{1}{2}$$

$$\text{we get on solving} \quad \cos^2 \omega = \frac{2}{3};$$

and $\cos \omega$ must be positive, for $0 < \omega < \frac{1}{2}\pi$.

But this will not satisfy our equation ; it satisfies

$$\frac{1}{8}\{8-5\cos^2\omega-\cos\omega(25\cos^2\omega-16)^{\frac{1}{2}}\}=\frac{1}{2}.$$

So $\frac{1}{8}\{8-5\cos^2\omega+\cos\omega(25\cos^2\omega-16)^{\frac{1}{2}}\}$ never $=\frac{1}{2}$, and it is sometimes greater, so it is always greater, since it is continuous.

So the second condition is never fulfilled.

The other condition is

$$x < 2 \sin^{-1} \frac{1}{8} \{8-5\cos^2\omega-\cos\omega(25\cos^2\omega-16)^{\frac{1}{2}}\}^{\frac{1}{2}} < \theta \quad \dots \quad (2)$$

This assumes $25\cos^2\omega-16 < 0$,

$$\therefore \cos\omega < \frac{4}{5},$$

i.e. $\omega < 36^\circ$ about, which will be true.

For this subcase to actually occur we must have

$$\theta > 2 \sin^{-1} \tan \omega.$$

We can see that this will always happen by considering fig. 1, for what we assert is that there are always real positions of F and G inside the figure DE ; and we can see that this is true, for ABD, A'B'D always cut, so that DE always exists, and is of finite size, even when F and G coincide in Y, so that F and G are inside the figure DE in this case, and cannot get outside it until FG has attained a certain finite size.

5. We will now consider the subcase when $2 \sin^{-1} \tan \omega < x < \theta$ and F and G lie inside DE, fig. 1. Then the conditions will be fulfilled if C lies outside the figure FG, the chance of which is

$$\frac{2\pi - \text{area FG}}{2\pi}.$$

And area FG = 2(sector FQG - ΔFQG)

$$\begin{aligned} &= 2 \{ (1 - \sin \omega) \text{F}\hat{\text{Q}}\text{G} + (\pi - \text{F}\hat{\text{Q}}\text{G} - 2\text{F}\hat{\text{G}}\text{Q}) \} \\ &= 2\pi - 2 \sin \omega \text{F}\hat{\text{Q}}\text{G} - 4\text{F}\hat{\text{G}}\text{Q} ; \end{aligned}$$

$\therefore$ chance of conditions being fulfilled = $\pi^{-1} (\sin \omega \text{F}\hat{\text{Q}}\text{G} + 2\text{F}\hat{\text{G}}\text{Q})$.

$$\text{F}\hat{\text{Q}}\text{G} = 2\text{Y}\hat{\text{Q}}\text{F}$$

$$\begin{aligned} &= 2 \cos^{-1} (\tan \text{QY} \cot \text{QF}) \\ &= 2 \cos^{-1} \{ \tan \omega \cot \cos^{-1} (\sin \omega \operatorname{cosec} \frac{1}{2}x) \} \\ &= 2 \cos^{-1} \left\{ \sin \omega \tan \omega \left(\sin^2 \frac{x}{2} - \sin^2 \omega \right)^{-\frac{1}{2}} \right\} \end{aligned}$$

and $\text{F}\hat{\text{G}}\text{Q} = \sin^{-1} (\sin \text{QY} \operatorname{cosec} \text{QF})$

$$= \sin^{-1} \left(\tan \omega \operatorname{cosec} \frac{x}{2} \right) ;$$

$\therefore$ chance of conditions being fulfilled

$$\begin{aligned} &= 2\pi^{-1} \left[\sin \omega \cos^{-1} \left\{ \sin \omega \tan \omega \left(\sin^2 \frac{x}{2} - \sin^2 \omega \right)^{-\frac{1}{2}} \right\} \right. \\ &\quad \left. + \sin^{-1} \left(\tan \omega \operatorname{cosec} \frac{x}{2} \right) \right] ; \end{aligned}$$

∴ the total chance of conditions being fulfilled under this subcase

$$= \int_0^{\theta} \frac{2\pi^{-1}}{2 \sin^{-1} \tan \omega} \left[\sin \omega \cos^{-1} \left\{ \sin \omega \tan \omega \left(\sin^2 \frac{x}{2} - \sin^2 \omega \right)^{-\frac{1}{2}} \right\} \right. \\ \left. + \sin^{-1} \left(\tan \omega \operatorname{cosec} \frac{x}{2} \right) \right] \sin x \, dx.$$

$$\begin{aligned} & \text{Now } \int \cos^{-1} \{ \sin \omega \tan \omega (\sin^2 \frac{1}{2}x - \sin^2 \omega)^{-\frac{1}{2}} \} \sin x \, dx \\ &= -\cos x \cos^{-1} \{ \sin \omega \tan \omega (\sin^2 \frac{1}{2}x - \sin^2 \omega)^{-\frac{1}{2}} \} \\ & \quad + \frac{1}{2} \sin \omega \tan \omega \int \cos x \sin \frac{1}{2}x \cos \frac{1}{2}x (\sin^2 \frac{1}{2}x - \sin^2 \omega)^{-1} \\ & \quad \quad \quad (\sin^2 \frac{1}{2}x - \tan^2 \omega)^{-\frac{1}{2}} \, dx \\ &= -\cos x \cos^{-1} \{ \sin \omega \tan \omega (\sin^2 \frac{1}{2}x - \sin^2 \omega)^{-\frac{1}{2}} \} \\ & \quad + 2^{-\frac{1}{2}} \sin \omega \tan \omega \int \sin x \cos x (\cos 2\omega - \cos x)^{-1} \\ & \quad \quad \quad (1 - 2 \tan^2 \omega - \cos x)^{-\frac{1}{2}} \, dx \\ & \quad \int \sin x \cos x (\cos 2\omega - \cos x)^{-1} (1 - 2 \tan^2 \omega - \cos x)^{-\frac{1}{2}} \, dx \\ & \quad \quad \quad = -\int \sin x (1 - 2 \tan^2 \omega - \cos x)^{-\frac{1}{2}} \, dx \\ & \quad + \cos 2\omega \int \sin x (\cos 2\omega - \cos x)^{-1} (1 - 2 \tan^2 \omega - \cos x)^{-\frac{1}{2}} \, dx \\ &= -2(1 - 2 \tan^2 \omega - \cos x)^{\frac{1}{2}} \\ & \quad + 2 \cos 2\omega \int (\cos 2\omega - 1 + 2 \tan^2 \omega + z^2)^{-1} \, dz, \\ & \quad \text{putting } 1 - 2 \tan^2 \omega - \cos x = z^2 \\ &= -2(1 - 2 \tan^2 \omega - \cos x)^{\frac{1}{2}} \\ & \quad + 2^{\frac{1}{2}} \operatorname{cosec} \omega \cot \omega \cos 2\omega \tan^{-1} (2^{-\frac{1}{2}} \operatorname{cosec} \omega \cot \omega z) \\ &= -2(1 - 2 \tan^2 \omega - \cos x)^{\frac{1}{2}} + 2^{\frac{1}{2}} \cos 2\omega \operatorname{cosec} \omega \cot \omega \\ & \quad \tan^{-1} \{ \operatorname{cosec} \omega \cot \omega 2^{-\frac{1}{2}} (1 - 2 \tan^2 \omega - \cos x)^{\frac{1}{2}} \} \\ &\therefore \int \cos^{-1} \{ \sin \omega \tan \omega (\sin^2 \frac{1}{2}x - \sin^2 \omega)^{-\frac{1}{2}} \} \sin x \, dx \\ &= -\cos x \cos^{-1} \{ \sin \omega \tan \omega (\sin^2 \frac{1}{2}x - \sin^2 \omega)^{-\frac{1}{2}} \} \\ & \quad - \sin \omega \tan \omega 2^{\frac{1}{2}} (1 - 2 \tan^2 \omega - \cos x)^{\frac{1}{2}} + \cos 2\omega \tan^{-1} \\ & \quad \quad \quad \{ \operatorname{cosec} \omega \cot \omega 2^{-\frac{1}{2}} (1 - 2 \tan^2 \omega - \cos x)^{\frac{1}{2}} \}; \\ &\therefore \int_0^{\theta} \frac{2 \sin \omega \pi^{-1} \cos^{-1} \{ \sin \omega \tan \omega (\sin^2 \frac{x}{2} - \tan^2 \omega)^{-\frac{1}{2}} \} \sin x \, dx}{2 \sin^{-1} \tan \omega} \\ &= 2 \sin \omega \pi^{-1} [-\cos \theta \cos^{-1} \{ \sin \omega \tan \omega (\sin^2 \frac{1}{2}\theta - \tan^2 \omega)^{-\frac{1}{2}} \} \\ & \quad - \sin \omega \tan \omega 2^{\frac{1}{2}} (1 - 2 \tan^2 \omega - \cos \theta)^{\frac{1}{2}} + \cos 2\omega \tan^{-1} \\ & \quad \quad \quad \{ 2^{-\frac{1}{2}} \operatorname{cosec} \omega \cot \omega (1 - 2 \tan^2 \omega - \cos \theta)^{\frac{1}{2}} \}] \end{aligned}$$

and

$$\begin{aligned}
 & \int \sin^{-1} (\tan \omega \operatorname{cosec} \tfrac{1}{2}x) \sin x dx = -\cos x \sin^{-1} (\tan \omega \operatorname{cosec} \tfrac{1}{2}x) \\
 & \quad - \int \tfrac{1}{2} \tan \omega \cos x \cos \tfrac{1}{2}x \operatorname{cosec}^2 \tfrac{1}{2}x (1 - \tan^2 \omega \operatorname{cosec}^2 \tfrac{1}{2}x)^{-\frac{1}{2}} dx \\
 & \quad - \int \tfrac{1}{2} \tan \omega \cos x \cos \tfrac{1}{2}x \operatorname{cosec}^2 \tfrac{1}{2}x (1 - \tan^2 \omega \operatorname{cosec}^2 \tfrac{1}{2}x)^{-\frac{1}{2}} dx \\
 & = \int \tan \omega (\tfrac{1}{2} \cos \tfrac{1}{2}x \operatorname{cosec}^2 \tfrac{1}{2}x - \cos \tfrac{1}{2}x) (1 - \tan^2 \omega \operatorname{cosec}^2 \tfrac{1}{2}x)^{-\frac{1}{2}} dx \\
 & = -\sin^{-1} (\tan \omega \operatorname{cosec} \tfrac{1}{2}x) - \int \sin \tfrac{1}{2}x \cos \tfrac{1}{2}x \tan \omega \\
 & \quad (\sin^2 \tfrac{1}{2}x - \tan^2 \omega)^{-\frac{1}{2}} dx \\
 & = -\sin^{-1} (\tan \omega \operatorname{cosec} \tfrac{1}{2}x) - 2 \tan \omega (\sin^2 \tfrac{1}{2}x - \tan^2 \omega)^{\frac{1}{2}}; \\
 \therefore \int \sin^{-1} (\tan \omega \operatorname{cosec} \tfrac{1}{2}x) \sin x dx & = 2 \sin^2 \tfrac{1}{2}x \sin^{-1} (\tan \omega \operatorname{cosec} \tfrac{1}{2}x) \\
 & \quad + 2 \tan \omega (\sin^2 \tfrac{1}{2}x - \tan^2 \omega)^{\frac{1}{2}}; \\
 \theta \\
 \therefore \int \frac{2\pi^{-1} \sin^{-1} (\tan \omega \operatorname{cosec} \tfrac{1}{2}x) \sin x dx}{2 \sin^{-1} \tan \omega} & = 4\pi^{-1} \sin^2 \tfrac{1}{2}\theta \sin^{-1} \\
 & \quad (\tan \omega \operatorname{cosec} \tfrac{1}{2}\theta) \\
 & \quad + 4\pi^{-1} \tan \omega \left(\sin^2 \tfrac{\theta}{2} - \tan^2 \omega \right)^{\frac{1}{2}} - 2 \tan^2 \omega;
 \end{aligned}$$

$\therefore$ total chance under this subcase is

$$\begin{aligned}
 & 2\pi^{-1} \sin \omega [\cos 2\omega \tan^{-1} \{2^{-\frac{1}{2}} \operatorname{cosec} \omega \cot \omega (1 - 2 \tan^2 \omega - \cos \theta)^{\frac{1}{2}}\} \\
 & \quad - \cos \theta \cos^{-1} \{\sin \omega \tan \omega (\sin^2 \tfrac{1}{2}\theta - \sin^2 \omega)^{-\frac{1}{2}}\} \\
 & - \sin \omega \tan \omega 2^{\frac{1}{2}} (1 - 2 \tan^2 \omega - \cos \theta)^{\frac{1}{2}}] + 4\pi^{-1} \sin^2 \tfrac{1}{2}\theta \sin^{-1} \\
 & \quad (\tan \omega \operatorname{cosec} \tfrac{1}{2}\theta) \\
 & + 4\pi^{-1} \tan \omega (\sin^2 \tfrac{1}{2}\theta - \tan^2 \omega)^{\frac{1}{2}} - 2 \tan^2 \omega \dots \dots \dots (3)
 \end{aligned}$$

6. If F and G lie outside the area common to ABD, A'B'D, (fig. 2), let ABD cut AFB' in H and A'FB in K, and let A'B'D cut AFB' in L and A'FB in M. Then the conditions will be satisfied if C falls outside HKML, and then only.

$$\begin{aligned}
 & \text{Now } AKHB + B'HLA + B'MLA' + A'KMB \\
 & = AFB + AFMD + MDK + FML + BFLE + LEH + HKML \\
 & + ADB' + DKGB' + HKG + MDK + AFMD + FML + HKML \\
 & + A'GB' + GHEA' + LEH + HKG + B'DKG + MDK + HKML \\
 & + A'EB + FLEB + FML + LEH + A'GHE + GHK + HKML \\
 & = 2\pi + FML + BFLE + LEH + HKML \\
 & \quad + MDK + AFMD + FML + HKML \\
 & \quad + HKG + B'DKG + MDK + HKML \\
 & \quad + LEH + A'GHE + GHK + HKML - HKML \\
 & = 2\pi + 4BFKE - HKML; \\
 \therefore HKML & = 2\pi + 4BFKE - 2AKHB + 2AHB';
 \end{aligned}$$

∴ chance of conditions being fulfilled

$$= \frac{2\pi - \text{HKML}}{2\pi} = \pi^{-1}(\text{AKHB} + \text{AFHB}' - 2\text{BFKE}).$$

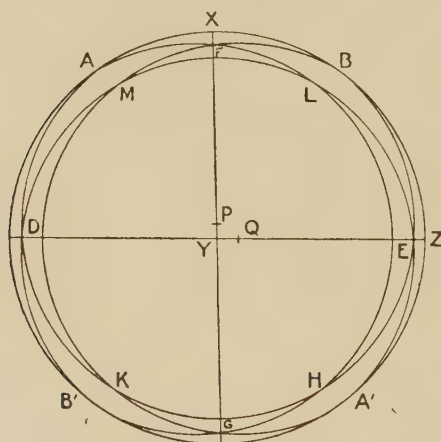


FIG. 2.

7. Now

$$\hat{PBA} = \cos^{-1}(\tan \frac{1}{2}x \tan \omega)$$

$$\hat{QBA}' = \cos^{-1}(\cot \frac{1}{2}x \tan \omega);$$

$$\begin{aligned} \therefore \hat{PBQ} &= \pi - \cos^{-1}(\tan \frac{1}{2}x \tan \omega) - \cos^{-1}(\cot \frac{1}{2}x \tan \omega) \\ &= \sin^{-1}(\tan \frac{1}{2}x \tan \omega) + \sin^{-1}(\cot \frac{1}{2}x \tan \omega), \end{aligned}$$

and

$$\hat{PBK} = \hat{KBM} - \hat{PBM}$$

$$= \frac{1}{2}\hat{HBM} - (\frac{1}{2}\pi - \hat{PBQ})$$

$$= \frac{1}{2}(\hat{QBM} + \hat{PBM} - \hat{PBQ}) - (\frac{1}{2}\pi - \hat{PBQ})$$

$$= \frac{1}{2}\hat{PBQ}$$

$$= \frac{1}{2} \{ \sin^{-1}(\tan \frac{1}{2}x \tan \omega) + \sin^{-1}(\cot \frac{1}{2}x \tan \omega) \}.$$

Also

$$\cot \hat{BPQ} = \tan \hat{PBK} \cos PB = \tan \hat{PBK} \sin \omega$$

and area

$$\text{BFKE} = 2 \text{ sector PBK} - 2\Delta \text{ PBK}$$

$$= 2(1 - \sin \omega) \times 2\hat{BPQ} - 2(2\hat{BPQ} + 2\hat{PBK} - \pi)$$

$$= 2\pi - 4\hat{PBK} - 4 \sin \omega \hat{BPQ}$$

$$\begin{aligned} &= 2\pi - 2 \sin^{-1}(\tan \frac{1}{2}x \tan \omega) - 2 \sin^{-1}(\cot \frac{1}{2}x \tan \omega) \\ &\quad - 4 \sin \omega \cot^{-1}[\sin \omega \tan \frac{1}{2} \{ \sin^{-1}(\cot \frac{1}{2}x \tan \omega) \\ &\quad \quad + \sin^{-1}(\tan \frac{1}{2}x \tan \omega) \}]. \end{aligned}$$

Also

$$\begin{aligned} \text{AKHB} &= \text{sector PAKHB} + \triangle \text{APB} \\ &= (1 - \sin \omega)(2\pi - 2x\hat{\text{PB}}) + 2x\hat{\text{PB}} + 2\text{A}\hat{\text{B}}\text{P} - \pi \\ &= \pi(1 - 2 \sin \omega) + 2 \sin \omega \sin^{-1}(\sin \tfrac{1}{2}x \sec \omega) \\ &\quad + 2 \cos^{-1}(\tan \tfrac{1}{2}x \tan \omega) \end{aligned}$$

Similarly

$$\begin{aligned} \text{AFHB}' &= \pi(1 - 2 \sin \omega) + 2 \sin \omega \sin^{-1}(\cos \tfrac{1}{2}x \sec \omega) \\ &\quad + 2 \cos^{-1}(\cot \tfrac{1}{2}x \tan \omega); \end{aligned}$$

$$\begin{aligned} \therefore \text{AKHB} + \text{AFAB}' - 2\text{BFKE} \\ &= -2\pi(1 + 2 \sin \omega) + 2 \sin \omega \sin^{-1}(\sin \tfrac{1}{2}x \sec \omega) \\ &\quad + 2 \cos^{-1}(\tan \tfrac{1}{2}x \tan \omega) + 2 \sin \omega \sin^{-1}(\cos \tfrac{1}{2}x \sec \omega) \\ &\quad + 2 \cos^{-1}(\cot \tfrac{1}{2}x \tan \omega) \\ &\quad + 8 \sin \omega \cot^{-1}[\sin \omega \tan \tfrac{1}{2}\{\sin^{-1}(\cot \tfrac{1}{2}x \tan \omega) \\ &\quad + \sin^{-1}(\tan \tfrac{1}{2}x \tan \omega)\}] \\ &= 2 \sin \omega \sin^{-1}(\sin \tfrac{1}{2}x \sec \omega) + 2 \sin \omega \sin^{-1}(\cos \tfrac{1}{2}x \sec \omega) \\ &\quad + 2 \sin^{-1}(\tan \tfrac{1}{2}x \tan \omega) + 2 \sin^{-1}(\cot \tfrac{1}{2}x \tan \omega) \\ &\quad - 8 \sin \omega \tan^{-1}[\sin \omega \tan \tfrac{1}{2}\{\sin^{-1}(\cot \tfrac{1}{2}x \tan \omega) \\ &\quad + \sin^{-1}(\tan \tfrac{1}{2}x \tan \omega)\}]; \end{aligned}$$

$\therefore$ total chance of conditions being fulfilled under this subcase

$$\begin{aligned} &= 2\pi^{-1} \int_0^\pi \left\{ \sin \omega \sin^{-1}(\sin \tfrac{1}{2}x \sec \omega) + \sin \omega \sin^{-1}(\cos \tfrac{1}{2}x \sec \omega) \right. \\ &\quad + \sin^{-1}(\tan \tfrac{1}{2}x \tan \omega) + \sin^{-1}(\cot \tfrac{1}{2}x \tan \omega) \\ &\quad - 4 \sin \omega \tan^{-1}[\sin \omega \tan \tfrac{1}{2}\{\sin^{-1}(\cot \tfrac{1}{2}x \tan \omega) \\ &\quad + \sin^{-1}(\tan \tfrac{1}{2}x \tan \omega)\}] \left. \right\} \sin x \, dx \quad \dots \quad (4) \end{aligned}$$

$$\begin{aligned} 8. \int \sin^{-1}(\cot \tfrac{1}{2}x \tan \omega) \sin x \, dx \\ &= -\cos x \sin^{-1}(\cot \tfrac{1}{2}x \tan \omega) \\ &\quad - \int \tfrac{1}{2} \cos x \tan \omega \operatorname{cosec}^2 \tfrac{1}{2}x (1 - \cot^2 \tfrac{1}{2}x \tan^2 \omega)^{-\frac{1}{2}} dx \\ &= -\cos x \sin^{-1}(\cot \tfrac{1}{2}x \tan \omega) \\ &\quad - \int \tfrac{1}{2} \tan \omega \cos x \operatorname{cosec} \tfrac{1}{2}x (\sin^2 \tfrac{1}{2}x - \tan^2 \omega \cos^2 \tfrac{1}{2}x)^{-\frac{1}{2}} dx \\ &\quad - \int \tfrac{1}{2} \tan \omega \cos x \operatorname{cosec} \tfrac{1}{2}x (\sin^2 \tfrac{1}{2}x - \tan^2 \omega \cos^2 \tfrac{1}{2}x)^{-\frac{1}{2}} dx \\ &= \int \tfrac{1}{2} \tan \omega \operatorname{cosec} \tfrac{1}{2}x (1 - \sec^2 \omega \cos^2 \tfrac{1}{2}x)^{-\frac{1}{2}} dx \\ &\quad - \int \tan \omega \sin \tfrac{1}{2}x (1 - \sec^2 \omega \cos^2 \tfrac{1}{2}x)^{-\frac{1}{2}} dx \\ &= \int \tfrac{1}{2} \tan \omega \operatorname{cosec} \tfrac{1}{2}x (1 - \sec^2 \omega \cos^2 \tfrac{1}{2}x)^{-\frac{1}{2}} dx \\ &\quad + 2 \sin \omega \sin^{-1}(\cos \tfrac{1}{2}x \sec \omega) \\ &= 2 \sin \omega \sin^{-1}(\cos \tfrac{1}{2}x \sec \omega) + \int \tan \omega (u^2 + \tan^2 \omega)^{-\frac{1}{2}} du; \end{aligned}$$

putting

$$\begin{aligned} 1 - \cos^2 \frac{1}{2}x \sec^2 \frac{1}{2}\omega &= u^2 \cos^2 \frac{1}{2}x \\ &= 2 \sin \omega \sin^{-1}(\cos \frac{1}{2}x \sec \omega) + \tan^{-1}(u \cot \omega) \\ &= 2 \sin \omega \sin^{-1}(\cos \frac{1}{2}x \sec \omega) + \tan^{-1}\{\cot \omega(\sec^2 \frac{1}{2}x \\ &\quad - \sec^2 \omega)^{-\frac{1}{2}}\}; \end{aligned}$$

$$\begin{aligned} \therefore \int \sin^{-1}(\cot \frac{1}{2}x \tan \omega) \sin x \, dx \\ &= -\cos x \sin^{-1}(\cot \frac{1}{2}x \tan \omega) - 2 \sin \omega \sin^{-1}(\cos \frac{1}{2}x \sec \omega) \\ &\quad - \tan^{-1}\{\cot \omega (\sec^2 \frac{1}{2}x - \sec^2 \omega)^{-\frac{1}{2}}\} \end{aligned}$$

$$\begin{aligned} \text{and } \int_{\theta}^{\pi} \sin^{-1}(\tan \frac{1}{2}x \tan \omega) \sin x \, dx \\ &= - \int_{\pi-\theta}^{\pi} \sin^{-1}(\cot \frac{1}{2}x \tan \omega) \sin x \, dx \end{aligned}$$

$$\begin{aligned} \therefore \int_{\theta}^{\pi} \{\sin^{-1}(\cot \frac{1}{2}x \tan \omega) + \sin^{-1}(\tan \frac{1}{2}x \tan \omega)\} \sin x \, dx \\ &= \int_{\theta}^{\pi-\theta} \sin^{-1}(\cot \frac{x}{2} \tan \omega) \sin x \, dx \\ &= \cos \theta \sin^{-1}(\tan \frac{1}{2}\theta \tan \omega) - 2 \sin \omega \sin^{-1}(\sin \frac{1}{2}\theta \sec \omega) \\ &\quad - \tan^{-1}\{\cot \omega(\operatorname{cosec}^2 \frac{1}{2}\theta - \sec^2 \omega)^{-\frac{1}{2}}\} \\ &\quad + \cos \theta \sin^{-1}(\cot \frac{1}{2}\theta \tan \omega) \\ &\quad + 2 \sin \omega \sin^{-1}(\cos \frac{1}{2}\theta \sec \omega) + \tan^{-1}\{\cot \omega(\sec^2 \frac{1}{2}\theta \\ &\quad - \sec^2 \omega)^{-\frac{1}{2}}\} \dots \dots \dots (5) \end{aligned}$$

Similarly

$$\begin{aligned} \int_{\theta}^{\pi} \{\sin^{-1}(\sin \frac{1}{2}x \sec \omega) + \sin^{-1}(\cos \frac{1}{2}x \sec \omega)\} \sin x \, dx \\ &= \int_{\theta}^{\pi-\theta} \sin^{-1}(\sin \frac{1}{2}x \sec \omega) \sin x \, dx \end{aligned}$$

$$\begin{aligned} \text{and } \int \sin^{-1}(\sin \frac{1}{2}x \sec \omega) \sin x \, dx \\ &= \int 2 \cos^2 \omega \phi \sin 2\phi d\phi \quad \text{if } \sin \frac{1}{2}x = \cos \omega \sin \phi \\ &= \cos^2 \omega \left(-\phi \cos 2\phi + \frac{\sin 2\phi}{2} \right) \\ &= \sin \frac{1}{2}x (\cos^2 \omega - \sin^2 \frac{1}{2}x)^{\frac{1}{2}} \\ &\quad - (\cos^2 \omega - 2 \sin^2 \frac{1}{2}x) \sin^{-1}(\sin \frac{1}{2}x \sec \omega) \end{aligned}$$

$$\begin{aligned}
 \therefore \int_0^{\pi} \{ \sin^{-1}(\sin \frac{1}{2}x \sec \omega) + \sin^{-1}(\cos \frac{1}{2}x \sec \omega) \} \sin x \, dx \\
 = \cos \frac{1}{2}\theta (\cos^2 \omega - \cos^2 \frac{1}{2}\theta)^{\frac{1}{2}} - (\cos^2 \omega - 2 \cos^2 \frac{1}{2}\theta) \\
 \sin^{-1}(\cos \frac{1}{2}\theta \sec \omega) - \sin^2 \frac{1}{2}\theta (\cos^2 \omega - \sin^2 \frac{1}{2}\theta)^{\frac{1}{2}} \\
 + (\cos^2 \omega - 2 \sin^2 \frac{1}{2}\theta) \sin^{-1}(\sin \frac{1}{2}\theta \sec \omega) \dots (6) \\
 \tan \frac{1}{2} \{ \sin^{-1}(\cot \frac{1}{2}x \tan \omega) + \sin^{-1}(\tan \frac{1}{2}x \tan \omega) \} \\
 = \cot \frac{1}{2} \{ \cos^{-1}(\cot \frac{1}{2}x \tan \omega) + \cos(\tan \frac{1}{2}x \tan \omega) \} \\
 = \{ 1 - (1 - \tan \frac{1}{2}x \tan \omega)^{\frac{1}{2}} (1 + \tan \frac{1}{2}x \tan \omega)^{-\frac{1}{2}} (1 - \tan \omega \cot \frac{1}{2}x)^{\frac{1}{2}} \\
 (1 + \cot \frac{1}{2}x \tan \omega)^{-\frac{1}{2}} \} \{ (1 - \tan \frac{1}{2}x \tan \omega)^{\frac{1}{2}} \\
 (1 + \tan \frac{1}{2}x \tan \omega)^{-\frac{1}{2}} + (1 - \cot \frac{1}{2}x \tan \omega)^{\frac{1}{2}} \\
 (1 + \cot \frac{1}{2}x \tan \omega)^{-\frac{1}{2}} \}^{-1} \\
 = 2 \{ (1 - \tan \frac{1}{2}x \tan \omega)^{\frac{1}{2}} (1 + \tan \frac{1}{2}x \tan \omega)^{-\frac{1}{2}} (1 + \cot \frac{1}{2}x \tan \omega)^{-1} \\
 - (1 - \cot \frac{1}{2}x \tan \omega)^{\frac{1}{2}} (1 + \cot \frac{1}{2}x \tan \omega)^{-\frac{1}{2}} (1 - \tan \frac{1}{2}x \tan \omega)^{-1} \} \\
 \{ (1 - \tan \frac{1}{2}x \tan \omega)(1 + \tan \frac{1}{2}x \tan \omega)^{-1} - (1 - \cot \frac{1}{2}x \tan \omega) \\
 (1 + \cot \frac{1}{2}x \tan \omega)^{-1} \}^{-1} \\
 = \cot \omega \left(\cot \frac{x}{2} - \tan \frac{x}{2} \right)^{-1} \{ (1 - \tan^2 \frac{1}{2}x \tan^2 \omega)^{\frac{1}{2}} \\
 - (1 - \cot^2 \frac{1}{2}x \tan^2 \omega)^{\frac{1}{2}} \} \\
 = 2^{-\frac{1}{2}} \operatorname{cosec} \omega \sec x \{ \sin \frac{x}{2} (\cos x + \cos^2 \omega)^{\frac{1}{2}} \\
 - \cos \frac{x}{2} (\cos 2\omega - \cos x)^{\frac{1}{2}} \} ;
 \end{aligned}$$

$$\begin{aligned}
 \therefore \int \tan^{-1} [\sin \omega \tan \frac{1}{2} \{ \sin^{-1}(\cot \frac{1}{2}x \tan \omega) + \sin^{-1}(\tan \frac{1}{2}x \tan \omega) \}] \\
 \sin x \, dx \\
 = \int \tan^{-1} [2^{-\frac{1}{2}} \sec x \{ \sin \frac{1}{2}x (\cos x + \cos 2\omega)^{\frac{1}{2}} - \cos \frac{1}{2}x (\cos 2\omega \\
 - \cos x)^{\frac{1}{2}} \}] \sin x \, dx \\
 = -\cos x \tan^{-1} [2^{-\frac{1}{2}} \sec x \{ \sin \frac{1}{2}x (\cos x + \cos 2\omega)^{\frac{1}{2}} \\
 - \cos \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} \}] \\
 + \int 2^{-\frac{1}{2}} [\sec x \{ \frac{1}{2} \cos \frac{1}{2}x (\cos 2\omega + \cos x)^{\frac{1}{2}} - \frac{1}{2} \sin \frac{1}{2}x \sin x \\
 (\cos x + \cos 2\omega)^{-\frac{1}{2}} + \frac{1}{2} \sin \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} - \frac{1}{2} \cos \frac{1}{2}x \sin x \\
 (\cos 2\omega - \cos x)^{\frac{1}{2}} \} + \sin x \sec^2 x \{ \sin \frac{1}{2}x (\cos 2\omega + \cos x) \\
 - \cos \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} \}] [1 + \frac{1}{2} \sec^2 x \{ \sin^2 \frac{1}{2}x \\
 (\cos x + \cos 2\omega) + \cos^2 \frac{1}{2}x (\cos 2\omega - \cos x) - \sin x \\
 (\cos^2 2\omega - \cos^2 x)^{\frac{1}{2}} \}] \cos x \, dx
 \end{aligned}$$

The integral in this expression =

$$\begin{aligned}
 \int 2^{\frac{1}{2}} \cos x \{ \cos \frac{1}{2}x (\cos 2\omega + \cos x)^{\frac{1}{2}} - \frac{1}{2} \cos x \cos \frac{1}{2}x (\cos 2\omega + \cos x)^{\frac{1}{2}} \\
 - \sin \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} - \sin \frac{1}{2}x \cos x (\cos 2\omega - \cos x)^{\frac{1}{2}} \\
 - \frac{1}{2} \sin \frac{1}{2}x \sin x \cos x (\cos 2\omega + \cos x)^{-\frac{1}{2}} - \frac{1}{2} \cos \frac{1}{2}x \sin x \cos x \\
 (\cos 2\omega - \cos x)^{-\frac{1}{2}} \} \cdot \{ \cos^2 x + \cos^2 2\omega - \sin x \\
 (\cos^2 2\omega - \cos^2 x)^{\frac{1}{2}} \}^{-1} dx
 \end{aligned}$$

$$\begin{aligned}
&= 2^{\frac{1}{2}} \int \left\{ \cos \frac{1}{2}x (\cos 2\omega + \cos x)^{\frac{1}{2}} - \cos x \cos \frac{1}{2}x \cos^2 \omega \right. \\
&\quad \left. (\cos 2\omega + \cos x)^{-\frac{1}{2}} - \sin \frac{1}{2}x (\cos 2\omega - \cos x) \right. \\
&\quad \left. - \cos x \sin \frac{1}{2}x \cos^2 \omega (\cos 2\omega - \cos x)^{-\frac{1}{2}} \right\} \\
&\quad \left\{ \cos \frac{1}{2}x (\cos 2\omega + \cos x)^{\frac{1}{2}} - \sin \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} \right\}^{-2} dx \\
&= 2^{\frac{1}{2}} \int \cos x \left\{ \cos \frac{1}{2}x (\cos 2\omega + \cos x)^{\frac{1}{2}} - \sin \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} \right\}^{-1} dx \\
&\quad - 2^{\frac{1}{2}} \cos^2 \omega \int \left\{ \cos \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} + \sin \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} \right. \\
&\quad \left. (\cos^2 2\omega - \cos^2 x)^{-\frac{1}{2}} \left\{ \cos \frac{1}{2}x (\cos 2\omega + \cos x)^{\frac{1}{2}} - \sin \frac{1}{2}x \right. \right. \\
&\quad \left. \left. (\cos 2\omega - \cos x)^{\frac{1}{2}} \right\}^{-2} \cos^2 x dx \right\} \\
&= 2^{\frac{1}{2}} \int \left\{ \cos \frac{1}{2}x (\cos 2\omega + \cos x)^{\frac{1}{2}} - \sin \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} \right\}^{-1} \cos x dx \\
&\quad - 2^{-\frac{1}{2}} \int \left\{ (\cos^2 2\omega - \cos^2 x)^{\frac{1}{2}} + \sin x \cos 2\omega \right\} (\cos^2 2\omega - \cos^2 x)^{-\frac{1}{2}} \\
&\quad \left\{ \cos \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} - \sin \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} \right\}^{-1} \cos x dx \\
&= 2^{-\frac{1}{2}} \int \left\{ \cos \frac{1}{2}x (\cos 2\omega + \cos x)^{\frac{1}{2}} - \sin \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} \right\}^{-1} \cos x dx \\
&\quad - 2^{-\frac{1}{2}} \int \sin x \cos x \cos 2\omega (\cos^2 2\omega - \cos^2 x)^{-\frac{1}{2}} \left\{ \cos \frac{1}{2}x \right. \\
&\quad \left. (\cos 2\omega + \cos x)^{\frac{1}{2}} - \sin \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} \right\}^{-1} \cos x dx \\
&= 2^{-\frac{1}{2}} \sec^2 \omega \int \left\{ \cos \frac{1}{2}x (\cos 2\omega + \cos x)^{\frac{1}{2}} + \sin \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} \right\} dx \\
&\quad - 2^{-\frac{1}{2}} \cos 2\omega \sec^2 \omega \int \sin x (\cos^2 2\omega - \cos^2 x)^{-\frac{1}{2}} \left\{ \cos \frac{1}{2}x \right. \\
&\quad \left. (\cos 2\omega + \cos x)^{\frac{1}{2}} + \sin \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} \right\} dx \\
&= 2^{-\frac{1}{2}} \sec^2 \omega \left\{ \sin \frac{1}{2}x (\cos 2\omega + \cos x)^{\frac{1}{2}} + \frac{1}{x} \int \sin \frac{1}{2}x \sin x \right. \\
&\quad \left. (\cos 2\omega + \cos x)^{-\frac{1}{2}} dx - \cos \frac{1}{2}x (\cos 2\omega - \cos x)^{-\frac{1}{2}} \right. \\
&\quad \left. + \frac{1}{2} \int \sin x \cos \frac{1}{2}x (\cos 2\omega - \cos x)^{-\frac{1}{2}} dx \right\} - 2^{-\frac{1}{2}} \cos 2\omega \sec^2 \omega \\
&\quad \left\{ \int \sin x \cos \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} dx \right. \\
&\quad \left. + \int \sin x \sin \frac{1}{2}x (\cos 2\omega + \cos x)^{\frac{1}{2}} dx \right\} \\
&= 2^{-\frac{1}{2}} \sec^2 \omega \left\{ \sin \frac{1}{2}x (\cos 2\omega + \cos x)^{\frac{1}{2}} - \cos \frac{1}{2}x (\cos 2\omega - \cos x)^{\frac{1}{2}} \right\} \\
&\quad + 2^{-\frac{1}{2}} \tan^2 \omega \left\{ \int \cos \frac{1}{2}x \sin x (\cos 2\omega - \cos x)^{-\frac{1}{2}} dx + \int \sin \frac{1}{2}x \sin x \right. \\
&\quad \left. (\cos 2\omega + \cos x)^{\frac{1}{2}} dx \right\} \\
&\quad \int \cos \frac{1}{2}x \sin x (\cos 2\omega - \cos x)^{-\frac{1}{2}} dx \\
&\quad = 2^{\frac{1}{2}} \int \cos^2 \frac{1}{2}x \sin \frac{1}{2}x (\cos^2 \omega - \cos^2 \frac{1}{2}x)^{-\frac{1}{2}} dx \\
&\quad = -2^{\frac{1}{2}} \cos^2 \omega \sin^{-1} (\cos \frac{1}{2}x \sec \omega) + 2^{\frac{1}{2}} \cos \frac{1}{2}x \\
&\quad (\cos^2 \omega - \cos^2 \frac{1}{2}x)^{-\frac{1}{2}}.
\end{aligned}$$

Similarly

$$\begin{aligned}
\int \sin \frac{1}{2}x \sin x (\cos 2\omega + \cos x)^{-\frac{1}{2}} dx &= 2^{\frac{1}{2}} \cos^2 \omega \sin^{-1} (\sin \frac{1}{2}x \sec \omega) \\
&\quad - 2^{\frac{1}{2}} \sin \frac{1}{2}x (\cos^2 \omega - \sin^2 \frac{1}{2}x)^{-\frac{1}{2}};
\end{aligned}$$

$$\begin{aligned} & \therefore \int_{\theta}^{\frac{\pi}{2}} \tan^{-1} \left[\sin \omega \tan \frac{1}{2} \left\{ \sin^{-1} (\cot \frac{1}{2} x \tan \omega) + \sin^{-1} (\tan \frac{1}{2} x \tan \omega) \right\} \right] \\ & \qquad \qquad \qquad \sin x \, dx \\ & = \cos \theta \tan^{-1} \left[2^{-\frac{1}{2}} \sec \theta \left\{ \sin \frac{1}{2} \theta (\cos \theta + \cos 2\omega)^{\frac{1}{2}} - \cos \frac{1}{2} \theta \right. \right. \\ & \qquad \qquad \qquad \left. \left. (\cos 2\omega - \cos \theta)^{\frac{1}{2}} \right\} \right] - 2^{-\frac{1}{2}} \left\{ \sin \frac{1}{2} \theta (\cos 2\omega + \cos \theta)^{\frac{1}{2}} - \cos \frac{1}{2} \theta \right. \\ & \qquad \qquad \qquad \left. (\cos 2\omega - \cos \theta)^{\frac{1}{2}} \right\} + \sin^2 \omega \left\{ \sin^{-1} (\cos \frac{1}{2} \theta \sec \omega) - \sin^{-1} \right. \\ & \qquad \qquad \qquad \left. (\sin \frac{1}{2} \theta \sec \omega) \right\} ; \end{aligned}$$

$\therefore$ from equations (4), (5), and (6), chance of conditions being fulfilled under this subcase,

$$\begin{aligned} & = 2\pi^{-1} \left\{ \sin \omega \cos \frac{1}{2} \theta (\cos^2 \omega - \cos^2 \frac{1}{2} \theta)^{\frac{1}{2}} - \sin \omega (\cos^2 \omega - 2 \cos^2 \frac{1}{2} \theta) \right. \\ & \qquad \sin^{-1} (\cos \frac{1}{2} \theta \sec \omega) - \sin \omega \sin \frac{1}{2} \theta (\cos^2 \omega - \sin^2 \frac{1}{2} \theta)^{\frac{1}{2}} + \sin \omega \\ & \qquad (\cos^2 \omega - 2 \sin^2 \frac{1}{2} \theta) \sin^{-1} (\sin \frac{1}{2} \theta \sec \omega) + \cos \theta \sin^{-1} (\tan \frac{1}{2} \theta \tan \omega) \\ & \qquad - 2 \sin \omega \sin^{-1} (\sin \frac{1}{2} \theta \sec \omega) - \tan^{-1} \left\{ \cot \omega (\operatorname{cosec}^2 \frac{1}{2} \theta - \sec^2 \omega)^{\frac{1}{2}} \right\} \\ & \qquad + \cos \theta \sin^{-1} (\cot \frac{1}{2} \theta \tan \omega) + 2 \sin \omega \sin^{-1} (\cos \frac{1}{2} \theta \sec \omega) + \tan^{-1} \\ & \qquad \left\{ \cot \omega (\sec^2 \frac{1}{2} \theta - \sec^2 \omega)^{\frac{1}{2}} \right\} - 4 \sin \omega \cos \theta \tan^{-1} \left[2^{-\frac{1}{2}} \sec \theta \left\{ \sin \frac{1}{2} \theta \right. \right. \\ & \qquad \qquad \qquad \left. \left. (\cos \theta + \cos 2\omega)^{\frac{1}{2}} - \cos \frac{1}{2} \theta (\cos 2\omega - \cos \theta)^{\frac{1}{2}} \right\} \right] + 2^{\frac{3}{2}} \sin \omega \\ & \qquad \left\{ \sin \frac{1}{2} \theta (\cos 2\omega + \cos \theta)^{\frac{1}{2}} - \cos \frac{1}{2} \theta (\cos 2\omega - \cos \theta)^{\frac{1}{2}} \right\} - 4 \sin^3 \omega \\ & \qquad \left\{ \sin^{-1} (\cos \frac{1}{2} \theta \sec \omega) - \sin^{-1} (\sin \frac{1}{2} \theta \sec \omega) \right\} \dots \dots \quad (7) \end{aligned}$$

9. $\therefore$ from equations (2), (3), and (7), total chance of conditions being fulfilled $= 2 \tan^2 \omega$

$$\begin{aligned} & + 2\pi^{-1} \sin \omega \left[\cos 2\omega \tan^{-1} \left\{ \operatorname{cosec} \omega \cot \omega (\sin^2 \frac{1}{2} \theta - \tan^2 \omega)^{\frac{1}{2}} \right\} \right. \\ & \qquad \qquad \qquad \left. - \cos \theta \cos^{-1} \left\{ \sin \omega \tan \omega (\sin^2 \frac{1}{2} \theta - \sin^2 \omega)^{-\frac{1}{2}} \right\} \right. \\ & \qquad \qquad \qquad \left. - 2 \sin \omega \tan \omega (\sin^2 \frac{1}{2} \theta - \tan^2 \omega)^{\frac{1}{2}} \right] \\ & + 4\pi^{-1} \sin^2 \frac{1}{2} \theta \sin^{-1} (\tan \omega \operatorname{cosec} \frac{1}{2} \theta) - 2 \tan^2 \omega \\ & \qquad \qquad \qquad + 4\pi^{-1} \tan \omega (\sin^2 \frac{1}{2} \theta - \tan^2 \omega)^{\frac{1}{2}} \\ & + 2\pi^{-1} \left\{ \sin \omega \cos \frac{1}{2} \theta (\cos^2 \omega - \cos^2 \frac{1}{2} \theta)^{\frac{1}{2}} - \sin \omega (\cos^2 \omega - 2 \cos^2 \frac{1}{2} \theta) \right. \\ & \qquad \qquad \qquad \left. \sin^{-1} (\cos \frac{1}{2} \theta \sec \omega) \right. \\ & \qquad \qquad \qquad \left. - \sin \omega \sin \frac{1}{2} \theta (\cos^2 \omega - \sin^2 \frac{1}{2} \theta)^{\frac{1}{2}} + \sin \omega (\cos^2 \omega - 2 \sin^2 \frac{1}{2} \theta) \sin^{-1} \right. \\ & \qquad \qquad \qquad \left. (\sin \frac{1}{2} \theta \sec \omega) \right. \\ & \qquad \qquad \qquad + \cos \theta \sin^{-1} (\tan \frac{1}{2} \theta \tan \omega) - 2 \sin \omega \sin^{-1} (\sin \frac{1}{2} \theta \sec \omega) - \tan^{-1} \\ & \qquad \qquad \qquad \left\{ \cot \omega (\operatorname{cosec}^2 \frac{1}{2} \theta - \sec^2 \omega)^{\frac{1}{2}} \right\} \\ & \qquad \qquad \qquad + \cos \theta \sin^{-1} (\cot \frac{1}{2} \theta \tan \omega) + 2 \sin \omega \sin^{-1} (\cos \frac{1}{2} \theta \sec \omega) \\ & \qquad \qquad \qquad + \tan^{-1} \left\{ \cot \omega (\sec^2 \frac{1}{2} \theta - \sec^2 \omega)^{\frac{1}{2}} \right\} \\ & \qquad \qquad \qquad - 4 \sin \omega \cos \theta \tan^{-1} \left[2^{-\frac{1}{2}} \sec \theta \left\{ \sin \frac{1}{2} \theta (\cos \theta + \cos 2\omega)^{\frac{1}{2}} - \cos \frac{1}{2} \theta \right. \right. \\ & \qquad \qquad \qquad \left. \left. (\cos 2\omega - \cos \theta)^{\frac{1}{2}} \right\} \right] + 2^{\frac{3}{2}} \sin \omega \left\{ \sin^{\frac{1}{2}} \theta (\cos 2\omega + \cos \theta)^{\frac{1}{2}} - \cos \frac{1}{2} \theta \right. \\ & \qquad \qquad \qquad \left. (\cos 2\omega - \cos \theta)^{\frac{1}{2}} \right\} - 4 \sin^3 \omega \left\{ \sin^{-1} (\cos \frac{1}{2} \theta \sec \omega) \right. \\ & \qquad \qquad \qquad \left. - \sin^{-1} (\sin \frac{1}{2} \theta \sec \omega) \right\} \end{aligned}$$

$$\begin{aligned}
&= 4\pi^{-1} \sin^2 \frac{\theta}{2} \sin^{-1} (\tan \omega \operatorname{cosec} \frac{1}{2} \theta) + 2\pi^{-1} \sin \omega (\cos 2\omega - \cos \theta) \\
&\quad \cos^{-1} \{2^{\frac{1}{2}} \sin \omega \tan \omega (\cos 2\omega - \cos \theta)^{-\frac{1}{2}}\} \\
&+ 2\pi^{-1} \sin \omega (\sin^2 \frac{1}{2} \theta - \tan^2 \omega)^{\frac{1}{2}} + 4\pi^{-1} \cos^2 \frac{1}{2} \theta \sin^{-1} (\tan \frac{1}{2} \theta \tan \omega) \\
&\quad - 4\pi^{-1} \sin^2 \frac{1}{2} \theta \sin^{-1} (\cot \frac{1}{2} \theta \tan \omega) \\
&+ 2\pi^{-1} \sin \omega (3 \cos^2 \omega - 2 \sin^2 \frac{1}{2} \theta) \sin^{-1} (\cos \frac{1}{2} \theta \sec \omega) - 2\pi^{-1} \\
&\quad (3 \cos^2 \omega - 2 \cos^2 \frac{1}{2} \theta) \sin^{-1} (\sin \frac{1}{2} \theta \sec \omega) \\
&+ 3\pi^{-1} 2^{\frac{1}{2}} \sin \omega \{\sin \frac{1}{2} \theta (\cos 2\omega + \cos \theta)^{\frac{1}{2}} - \cos \frac{1}{2} \theta (\cos 2\omega - \cos \theta)^{\frac{1}{2}}\} \\
&\quad - 8\pi^{-1} \cos \theta \sin \omega \tan^{-1} [2^{-\frac{1}{2}} \sec \theta \{\sin \frac{1}{2} \theta (\cos 2\omega + \cos \theta)^{\frac{1}{2}} \\
&\quad - \cos \frac{1}{2} \theta (\cos 2\omega - \cos \theta)^{\frac{1}{2}}\}].
\end{aligned}$$

10. We will now find the expansion of this in terms of ω as far as ω^3 .

First $\theta = 2 \sin^{-1} [\frac{1}{8} \{8 - 5 \cos^2 \omega - \cos \omega (25 \cos^2 \omega - 16)^{\frac{1}{2}}\}]^2$ from the inequality (1)

$$\begin{aligned}
&= 2 \sin^{-1} \left\{ \frac{1}{8} \omega^2 \left(\frac{32}{3} - \frac{64\omega^2}{27} \right) \right\}^{\frac{1}{2}} q.p. \\
&= 2 \sin^{-1} \left\{ 3^{-\frac{1}{2}} 2\omega \left(1 - \frac{1}{9} \omega^2 \right) \right\} q.p. \\
&= 4 \cdot 3^{-\frac{1}{2}} \omega \left(1 + \frac{1}{9} \omega^2 \right) q.p.
\end{aligned}$$

$$\text{So } \sin^{-1} (\tan \omega \operatorname{cosec} \frac{1}{2} \theta) = \sin^{-1} \{2 \cdot 3^{\frac{1}{2}} (1 + \frac{4}{9} \omega^2)\} q.p.$$

$$= \frac{1}{3} \pi + 4 \cdot 3^{-\frac{3}{2}} \omega^2 q.p. ;$$

$$\therefore \text{the first term} = \frac{1}{9} \omega^2 q.p.$$

$$\text{Also } \cos 2\omega - \cos \theta = (1 - 2\omega^2) - \left(1 - \frac{\theta^2}{2}\right) q.p.$$

$$= \frac{2}{3} \omega^2 q.p.$$

$$\text{So that } \cos^{-1} \{2^{\frac{1}{2}} \sin \omega \tan \omega (\cos 2\omega - \cos \theta)^{-\frac{1}{2}}\}$$

$$= \cos^{-1} (3^{\frac{1}{2}} \omega) q.p.$$

$$= \frac{\pi}{2} - \text{terms containing } \omega ;$$

$$\therefore \text{the second term} = \frac{2}{3} \omega^3 q.p.$$

$$\sin^2 \frac{1}{2} \theta - \tan^2 \omega = \frac{\omega^2}{3} q.p. ;$$

$$\therefore \text{the third term} = 4 \cdot 3^{-\frac{1}{2}} \pi^{-1} \omega^2$$

$$\sin^{-1} \left(\tan \frac{\theta}{2} \tan \omega \right) = \sin^{-1} (2 \cdot 3^{-\frac{1}{2}} \omega^2) ;$$

$$= 2 \cdot 3^{-\frac{1}{2}} \omega^2 q.p. ;$$

$$\therefore \text{the fourth term} = 8 \cdot 3^{-\frac{1}{2}} \cdot \pi^{-1} \omega^2$$

$$\sin^{-1} (\cot \frac{1}{2} \theta \tan \omega) = \sin^{-1} \frac{1}{2} \{3^{-\frac{1}{2}} (1 - \frac{2}{9} \omega^2)\} q.p.$$

$$= \frac{\pi}{3} - \text{terms in } \omega^2 ;$$

$$\begin{aligned}\therefore \text{the fifth term} &= \frac{1}{9} \omega^2 q.p. \\ 3 \cos^2 \omega - 2 \sin^2 \frac{1}{2} \theta &= 3 - \frac{1}{3} \omega^2 q.p. \\ \sin^{-1} (\cos \frac{1}{2} \theta \sec \omega) &= \sin^{-1} (1 - \frac{1}{6} \omega^2) q.p. \\ &= \frac{1}{2} \pi - 3^{-\frac{1}{2}} \omega + \text{terms in } \omega^3 ;\end{aligned}$$

$$\begin{aligned}\therefore \text{the sixth term} &= 3\omega - 2 \cdot 3^{\frac{1}{2}} \cdot \pi^{-1} \omega^2 - \frac{3}{8} \omega^2 \\ 3 \cos^2 \omega - 2 \cos^2 \frac{1}{2} \theta &= 1 - \frac{\omega^2}{3} q.p. \\ \sin^{-1} (\sin \frac{1}{2} \theta \sec \omega) &= \sin^{-1} (2 \cdot 3^{-\frac{1}{2}} \omega) q.p. \\ &= 2 \cdot 3^{-\frac{1}{2}} \omega q.p. ;\end{aligned}$$

$$\begin{aligned}\therefore \text{the seventh term} &= 4 \cdot 3^{-\frac{1}{2}} \pi^{-1} \omega^2 \\ \cos 2\omega + \cos \theta &= 2 + \text{terms in } \omega^2 ; \\ \therefore \sin \frac{1}{2} \theta (\cos 2\omega + \cos \theta)^{\frac{1}{2}} - \cos \frac{1}{2} \theta (\cos 2\omega - \cos \theta)^{\frac{1}{2}} \\ &= 3^{-\frac{1}{2}} \omega + \text{terms in } \omega^3 ;\end{aligned}$$

$$\begin{aligned}\therefore \text{the eighth term} &= 2 \cdot 3^{\frac{1}{2}} \cdot \pi \omega^2 q.p. \\ \text{and } \tan^{-1} \{ \sin \frac{1}{2} \theta (\cos 2\omega + \cos \theta)^{\frac{1}{2}} - \cos \frac{1}{2} \theta (\cos 2\omega - \cos \theta)^{\frac{1}{2}} \} \\ &= \frac{\omega}{\sqrt{3}} + \text{terms in } \omega^3 ;\end{aligned}$$

$$\therefore \text{the ninth term} = 8 \cdot 3^{-\frac{1}{2}} \cdot \pi^{-1} \omega^3 q.p.$$

Collecting these terms and giving them their proper signs we get total chance

$$= 3\omega - \frac{1}{2} \omega^3 q.p.$$

II. This is with the inradius expressed in radians.

Putting ω radians = δ degrees

$$\begin{aligned}\text{we get chance} &= \frac{1}{60} \pi \delta - \frac{11}{2 \times 180^3} \pi^3 \delta^3 \\ &= .0524 \delta - .000029 \delta^3 q.p.\end{aligned}$$

Putting $\delta = 4$ we get chance = .202 $q.p.$;

putting $\delta = 1$ we get chance = .052 $q.p.$

12. That the paths of three meteors should appear to meet in a point is not, however, a sufficient condition for their appearing to give a radiant. It is also necessary, (a) that the points at which they first appear should all be within 90° of one of the two points where their paths appear to meet, (b) that they should all be proceeding away from this point of meeting of their paths. Assuming that one direction is as probable as the other for each meteorite, the chance of the second of these conditions being fulfilled is clearly $\frac{1}{8}$.

13. To find the probability of the first being fulfilled let the zenith distance of P, one of the points of intersection of their

paths be z (fig. 3), then the area of that portion of the visible half of the celestial sphere which is within 90° of P is clearly $2\pi - 2z$.

The case now begins to present some difficulty, but, as a simple assumption, I will take a uniform distribution of points of intersection P , and also a uniform distribution of starting points. The former of these clearly agrees with my former assumption of a uniform distribution of tracks.

Then the chance of any one of the starting points of meteorites falling within 90° of P will be $\pi^{-1}(\pi - z)$, and the chance of the

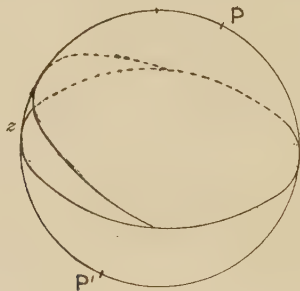


FIG. 3.

three falling within 90° of P is $\pi^{-3}(\pi - z)^3$, and the chance of the zenith distance of P lying between z and $z + dz = \sin z \, dz$.

$\therefore$ the total chance of the three points of first appearance lying within 90° of P , the meeting point of their paths above the horizon, $= \int_0^{\frac{\pi}{2}} \pi^{-3}(\pi - z)^3 dz$.

Besides this there is the possibility of their lying within 90° of P' , the point antipodal to P .

The chance of this is in a similar fashion $= \int_0^{\frac{\pi}{2}} \pi^{-3} z^3 dz$.

$\therefore$ the total chance of condition (a) being fulfilled

$$\begin{aligned} &= \int_0^{\frac{\pi}{2}} \pi^{-3} z^3 \sin z \, dz + \int_0^{\frac{\pi}{2}} \pi^{-3} (\pi - z)^3 \sin z \, dz \\ &= \int_0^{\pi} \pi^{-3} z^3 \sin z \, dz \\ &= \pi^{-3} \pi \left[-z^3 \cos z + 3z^2 \sin z + 6z \cos z - 6 \sin z \right] \\ &= \pi^{-2}(\pi^2 - 6). \end{aligned}$$

14. $\therefore$ the chance of all three conditions being fulfilled and the three meteors appearing to indicate a radiant

$$\begin{aligned} &= .202 \times \frac{1}{8} \times \pi^{-2}(\pi^2 - 6) = .0995 = \frac{1}{10.1} q.p. \text{ if } \delta = 4 \\ &= .052 \times \frac{1}{8} \times \pi^{-2}(\pi^2 - 6) = .0256 = \frac{1}{39.1} q.p. \text{ if } \delta = 1. \end{aligned}$$

15. According to these results the process of determining a radiant from three meteors is in general sound, though the chance of at least one radiant of this sort being found among several meteors may be considerable. This and similar questions I may profitably leave to those of a greater practical acquaintance with the subject.

On the Variable Star Y Aurigæ (Ch. 1929).

By A. Stanley Williams.

$$\begin{array}{l} \text{h m s} \\ \text{R.A.} = 5 \ 18 \ 20, \text{Decl.} = +42^{\circ} \ 18' \ 5 \ (1855) \\ \text{,,} = 5 \ 21 \ 32, \text{,,} = +42 \ 21' \ 1 \ (1900) \end{array}$$

When the unknown period of a variable star is longer than a few days there is usually little difficulty in ascertaining its real length, since both this and the form of the light curve can generally be derived directly from the observations made night by night. There is likewise not usually any very great difficulty when the period is much less than a day, for then the variation is so rapid that the observations of a few consecutive hours are sufficient to indicate the correct period. It is when the period falls between about one day and four or five days that the most difficulty arises, and the difficulty is intensified when the increase in brightness is much more rapid than the decrease, since in such cases a marked change in the brightness of the star may be noticed in a few hours, giving rise to the impression that the period is a much shorter one than it really is. The variable *Y Aurigæ* belongs to this last-mentioned class. In the *Astronomische Nachrichten*, No. 3708, the period of this star was stated by the writer to be $0^{\text{d}}.7925$, this period satisfying all the data at the time available, whilst some observations made during the increase seemed to show that the period must be a very short one, there being a considerable increase in brightness in the course of three or four hours. But recently, upon reducing the observations of this star made here during the past four years, it was found that this very short period failed to satisfy them for any very long interval of time; nor could any variation of the same very short period be made to do so. On the other hand, a period of $3^{\text{d}}.862$, or thereabouts, seemed to satisfactorily represent all the observations. The present paper contains a discussion of the observations made here during the last four years, and in order that the whole of the evidence as to the real period and light curve may be accessible, the actual observations have been stated in full.

The observations in question were all made with a power of 75 on a $2\frac{3}{4}$ -inch refractor, and in order to eliminate the effects

of "position error" the observer's head was always so held that the eyes were parallel to a line joining the two stars BD. $+42^{\circ} 1298$ and BD. $+42^{\circ} 1291$, the last-mentioned star being to the left. The comparison stars used and the adopted light scales and magnitudes are given in the following table:—

TABLE I.

Comparison Stars.

Star.	Name.	BD. Mag.	Light Scale.			Adopted Magnitude.
			$a-1.$	$a-2.$	$b.$	
<i>h</i>	BD. $+42^{\circ} 1291$	8.2	...	27.4	31.6	8.19
<i>l</i>	BD. $+42^{\circ} 1305$	9.0	15.4	23.4	...	8.47
<i>f</i>	BD. $+42^{\circ} 1301$	9.0	12.1	17.7	21.9	8.85
<i>b</i>	BD. $+42^{\circ} 1297$	9.5	6.4	7.9	7.2	9.52
<i>c</i>	BD. $+42^{\circ} 1302$	9.4	5.0	5.0	5.0	9.72

In the 4th, 5th, and 6th columns of the above table are three distinct light scales, that headed $a-1$ being derived from the observations of 1901, and that $a-2$ from the observations of 1902 and 1903. The light scale b has been added for the purpose of comparison only. It was derived from sequences of steps made on seven nights in 1902-3, the head being held so that the eyes were parallel to a line joining the two stars undergoing comparison,* instead of with the eyes parallel to the two stars referred to above. It will be seen that the two light scales $a-1$ and $a-2$ differ widely from each other; and it is clear that a great change must have occurred in the observer's habit of observing between 1901 and 1902-3, the value of a step having diminished by nearly a half. The exact step values are $0^{\text{m}}.120$ for 1901, and $0^{\text{m}}.068$ for 1902-3. No sensible change seems to have occurred, however, apart from this alteration of step value, the ratio between the brightnesses of the stars remaining unaltered. The last column of the above table gives the adopted magnitudes of the comparison stars.

Table II. contains all the original observations. In the 6th column will be found the deduced brightness of the variable, expressed according to the light scale $a-1$ for the observations of 1901, and according to the scale $a-2$ for the subsequent observations. The last column gives the corresponding magnitude. The observations of 1904 were reduced by means of the light scale $a-2$, though they had not entered into the formation of that scale.

* This is the method of observing at present employed by the writer in observing all variable stars excepting *Y Aurigæ*, as it appears that observations made in this way are, so far as he is concerned, practically free from the effects of "position error." This might not, however, be the case with the observations of another observer.

TABLE II.
Observations.

No.	Date.	G.M.T.	J.D. 241.	Observations.	Bright- ness ($a-1$).	Mag.
	1901.	h m				
1	Feb. 6	15 30	5422.65	v 2 b, v 2 c, f 5 v, l 8 v	7.5	9.40
2	11	13 15	5427.55	b 1 v o c	5.1	9.69
3	13	13 00	5429.54	v 3 b, v 3 c, f 2 v, l 5 v	9.4	9.17 (1)
4	20	12 45	5436.53	v 7 b, v 9 c, v o f	12.7	8.78
5	Mar. 1	12 15	5445.51	v 1 b, v 3 c	7.7	9.38
6	3	13 00	5447.54	v 1 f, v 6 b, v 7 c	12.5	8.80
7	12	11 40	5456.49	l 5 v 5 b, f 1 v 6 c	11.1	8.97 (2)
8	13	12 15	5457.51	b 2 v	4.4	9.77 (3)
9	18	11 10	5462.47	b o v i c	6.3	9.55
10	21	12 45	5465.53	b 1 v o c	5.1	9.69
11	22	10 30	5466.44	b o v 2 c	6.5	9.52
12	24	11 00	5468.46	v 1 b, v 2 c	7.2	9.44 (4)
13	25	11 30	5469.48	b o v i c	6.3	9.55
14	26	10 10	5470.42	v 3 b, v 4 c	9.2	9.20
15		11 55	5470.50	f 1 v 5 b, v 6 c	11.2	8.96
16		12 55	5470.54	f o v 6 b, v 7 c	12.2	8.84
17	27	10 10	5471.42	v 1 f, v 6 b	12.8	8.77
18		11 55	5471.50	v 1 f, v 6 b	12.8	8.77 (5)
19	28	11 00	5472.46	f 3 v 2 b, v 3 c	8.6	9.27
20		12 30	5472.52	f 4 v 2 b, v 3 c	8.2	9.32
21	29	10 15	5473.43	b o 5 v o c, f 6 v	5.3	9.67 (6)
22	31	10 00	5475.42	f o $\pm$ v 5 b	11.4	8.93 (7)
23	Apr. 1	10 00	5476.42	f 3 v 2 b, v 3 c	8.6	9.27
24		12 00	5476.50	f 5 v o 5 b	6.9	9.47
25	4	9 50	5479.41	v 3 b	9.4	9.17 (8)
26	12	9 50	5487.41	f 1 v 4 b, v 5 c	10.7	9.02
27	13	10 30	5488.44	b 1 v	5.4	9.65
28	14	9 05	5489.38	b 1 v	5.4	9.65
29	15	10 00	5490.42	v 2 f, v 7 b	13.7	8.66
30	17	9 50	5492.41	b 1 v	5.4	9.65 (9)
31	18	9 40	5493.40	v o $\pm$ b	6.4 $\pm$ 9.53 $\pm$	(10)
32	19	9 45	5494.41	v 1 f, v 6 b	12.7	8.78
33	20	9 45	5495.41	f 3 v 2 b, v 5 c	8.7	9.26
34	21	9 00	5496.38	b o v 2 c	6.7	9.50
35	22	9 25	5497.39	v o b	6.4	9.53
36		9 38	5497.40	v 1 b	7.4	9.41

No.	Date.	G.M.T.	J.D. 241.	Observations.	Bright- ness (a-1).	Mag.
	1901.	h m				
37	Apr. 22	10 00	5497.42	v 2 b	8.4	9.29 (10)
38		10 25	5497.43	v 3 b	9.4	9.17 (11)
39	23	9 00	5498.38	v 2 f, v 6 b	13.3	8.71
40		11 00	5498.46	v 0.5 f, v 5 b	12.0	8.86
41	24	9 30	5499.40	v 2 b	8.4	9.29 (12)
42	25	9 35	5500.40	v 0 b	6.4	9.53
43	26	9 10	5501.38	v 1 b, v 3 c	7.7	9.38 (13)
44		9 25	5501.39	f 2 v 3 b, v 5 c	9.8	9.13
45		9 50	5501.40	f 0 v 4 b	11.2	8.96
46		10 00	5501.42	f 0.5 v 5 b	11.5	8.92
47		10 20	5501.43	v 0.5 f, v 6 b	12.5	8.80
48		10 35	5501.44	v 2 f, v 7 b	13.7	8.66
49		11 00	5501.46	v 2 f, v 7 b	13.7	8.66 (14)
50	28	8 50	5503.37	v 3.5 b	9.9	9.11
51	May 3	9 00	5508.38	b 1 v	5.4	9.65
52	5	9 00	5510.38	f 0 v 5 b	11.7	8.90
53	6	9 35	5511.40	v 0 b	6.4	9.53
54	13	10 00	5518.42	f 1 v 4 b	10.9	8.99
55	14	9 50	5519.41	f 5 v 0.5 b, v 3 c	7.1	9.45
56	15	9 50	5520.41	b 0.5 v	5.9	9.59
57	17	10 00	5522.42	b 1 v	5.4	9.65 (15)
58	18	10 00	5523.42	v 0 b	6.4	9.53
59	20	9 55	5525.41	f 3 v 7 b	14.2	8.60
60	21	9 50	5526.41	f 3 v 2 b	8.7	9.26
61	22	10 05	5527.42	v 0 b	6.4	9.53
62	23	10 00	5528.42	v 0.5 b	6.9	9.47
63	24	10 10	5529.42	v 6 b	12.4	8.81 (16)
64	Aug. 6	13 53	5603.58	f 6 v, b 1 v 1 c	5.7	9.62
65	8	11 50	5605.49	f 2 v 3 b	9.8	9.13
66	10	12 55	5607.54	f 3 v 2 b, v 2 c	8.5	9.26
67	11	14 30	5608.60	f 6 v 0 b, v 0.5 c	5.8	9.61
68	15	11 30	5612.48	b 1 v 0 c, f 6 v	5.5	9.64
69	16	11 40	5613.49	v 1.5 f, v 7 b, v 8 c	13.3	8.71
70	18	11 05	5615.46	f 4 v 1 b, v 1 c	7.4	9.41
71		14 00	5615.58	f 6 v 0 b, v 1 c	6.2	9.56
72	19	11 25	5616.48	f 5 v 0 b	6.5	9.52
73		14 15	5616.59	f 6 v 0.5 b, v 2 c	6.7	9.50
74	20	11 15	5617.47	v 3 f, v 9 b, v 11 c	15.5	8.44

No.	Date.	G.M.T.	J.D. 241.	Observations.	Bright- ness ($\alpha-1$).	Mag.
	1901.	h m				
75	Aug. 21	11 17	5618.47	f 3 v 3 b, (v 6 c)	9.3	9.19 (17)
76		14 15	5618.59	f 4.5 v 2.5 b, (v 7 c)	8.3	9.31
77	22	12 c8	5619.51	f 5.5 v o b, v 2 c	6.8	9.49
78	23	11 35	5620.48	b 1 v 3 c, f 7 v	6.1	9.57
79		14 15	5620.59	b 1 v, (v 5 c), f 7 v	5.3	9.67
80	24	13 25	5621.56	v 2.5 f, v 8.5 b	14.7	8.54
81	26	11 50	5623.49	v o b	6.4	9.53
82	27	14 10	5624.59	f 1 v 5 b	11.2	8.96
83	Sept. 1	11 05	5629.46	v 2 f, v 7 b	13.8	8.65 (18)
84	3	10 40	5631.44	f 3 v 3 b	9.3	9.19
85	4	10 10	5632.42	f 5 v o b	6.5	9.52
86	5	10 20	5633.43	f 2 v 3 b	9.8	9.13 (19)
87		11 18	5633.47	v 1 f, v 7 b	13.2	8.72 (19)
88		12 32	5633.52	v 3 f, v 8 b	14.8	8.53
	1902.				Scale $\alpha-2$.	
89	Mar. 4	12 25	5813.52	f 9 v, b 1 v	8.8	9.45
90		5 11 50	5814.49	v 2 f, v 10 b, v 12 c	18.8	8.78
91		6 12 15	5815.51	f 5 v 4.5 b, v 6 c	12.3	9.22
92	Apr. 3	10 10	5843.42	v o b	7.9	9.52 (20)
93		6 10 45	5846.45	f 6 v 3 b, v 5 c	11.0	9.31
94	24	9 15	5864.39	v 3 f, v 9 b, v 12 c	18.8	8.78
95	27	9 45	5867.41	b 1.5 v 2 c, f 10 v	6.9	9.58
96	28	9 15	5868.39	v 11 b 2 c, v 1 f	18.7	8.78
97		10 18	5868.43	v 11 b 2 c, v 1 f	18.7	8.78
98	July 28	13 00	5959.54	b 1 v	8.9	9.45
99	Aug. 24	14 55	5986.62	b 2 v 3 c	6.8	9.59
100		25 13 07	5987.55	f 2 v 7 b, v 12 c	16.8	8.91
101		27 12 55	5989.54	f 10 v 3 b, v 6 c	10.0	9.37
102		28 14 35	5990.61	b 1 v 4 c, f 10 v	7.6	9.54
103	Sept. 6	12 10	5999.51	v 3 f, v 13 b	20.7	8.65
104		7 11 10	6000.47	f 3 v 6 b	14.4	9.07
105		8 12 17	6001.51	b o v 4 c	8.3	9.49
106		26 11 15	6019.47	f o v 9 b, v 12 c	17.3	8.88
	1903.					
107	Jan. 28	15 35	6143.65	f 5 v 7 b	13.7	9.12
108	Mar. 21	12 27	6195.52	b 1 v 2 c	7.0	9.58
109		22 11 27	6195.48	f 2 v 9 b	16.1	8.96
110		26 11 10	6200.47	f 4 v 10 b	15.3	9.01
111	Apr. 22	10 35	6227.44	f 2 v 8 b, v 9 c	15.5	9.00

No.	Date.	G.M.T.		J.D. 241.	Observations.	Bright- ness (a-1).	Mag.
	1904.	h	m				
112	Mar.	8	11 55	6548.50	f 4 v 8 b, v 9 c	14.4	9.07 (20)
113		10	12 30	6550.52	b 1 v 2 c	7.0	9.58
114		22	11 45	6562.49	b 2 v 1 c	6.0	9.65
115			12 45	6562.53	b 0 v 3 c	7.9	9.52
116	Apr.	1	9 55	6572.41	f 7 v 2 b, v 4 c	10.0	9.37
117		6	10 00	6577.42	b 1 v 2 c	7.0	9.58
118		9	10 00	6580.42	f 10 v 0 b, v 2 c	7.7	9.53
119		10	10 20	6581.43	b 2 v 0 c	5.2	9.70
120			11 50	6581.49	v 0 b	7.9	9.52
121		11	10 00	6582.42	v 2 f, v 12 b	19.8	8.70
122		12	10 00	6583.42	f 6 v 4 b	11.8	9.25
123		13	10 00	6584.42	b 1 v 1 c	6.5	9.62 (21)
124		15	9 50	6586.41	v 1 f, v 11 b	18.1	8.78 (22)
125		16	10 05	6587.42	f 7 v 3 b, v 6 c	10.9	9.31
126		17	10 03	6588.42	b 2 v 1 c	6.0	9.65 (23)
127		18	9 20	6589.39	b 2 v 0 c	5.2	9.70
128			11 00	6589.46	f 10 b 0 v 4 c	7.7	9.53 (24)
129		20	9 40	6591.40	f 7 v 3 b, v 5 c	10.6	9.33

Notes.—(1) Clouds. (2) Not good, hazy. Var. seemed reddish or orange-coloured. (3) Thin cloud. (4) Hazy. (5) Poor obs., hazy and sky very bright. (6) Clouds. (7) Cirrus cloud. (8) Not satisfactory, sky very bright. (9) Hazy. (10) Cloud. (11) Thin cloud. (12) Thin cloud. (13) Probably an underestimate. (14) Not good, clouds. (15) Thin cloud. (16) Rough obs., thin cloud. (17) c seems very faint to-night. (18) Poor obs. (19) Not good. (20) Cloud directly after. (21) Misty cloud about. (22) Considerable thin cloud. (23) Some thin cloud. (24) Unsatisfactory, altitude low and stars unsteady.

The clearness of the sky and the interference from moonlight have always been recorded during the last two years ; but as this was not done in the case of the earlier observations, the particulars have been omitted from the above table. It may be stated, however, generally that moonlight seems to exercise very little influence upon the observations of the writer, and that all the cases in which the observations may have been seriously affected by the state of the sky have been mentioned in the foregoing notes. Except where otherwise mentioned, the observations may be regarded as satisfactory. In ordinary variable star work observations are usually not made under very unfavourable circumstances, but at first, when the nature of the variation was quite unknown, the star was observed whenever visible. This accounts for the relatively large number of observations that are mentioned as being certainly or possibly affected by cloud, &c.

A preliminary investigation having shown that the observa-

tions were nearly satisfied by a period of $3^d.862$, an ephemeris was computed from the following elements (Elements I.) :—

$$\text{Maximum} = \text{J.D. } 2415420.354 + 3^d.862 \text{ E}$$

and the observations were formed into seven groups and plotted upon transparent squared paper according to the intervals by which they followed the computed times of maximum. Three of the resulting charts were then superimposed one upon the other, and a provisional mean light curve (A) carefully drawn. By means of this provisional mean light curve the times of seven mean or normal maxima were then determined from the observations for a date corresponding approximately to the middle of each group. The details of these mean maxima are given in Table III.

TABLE III
Provisional Mean Maxima.

E.	Mean Obs. Maximum.	Computed Maximum (II.).	O—C. d	No. of Obs.
9	J.D. 2415455.347	5455.371	—0.024	33
24	5513.373	5513.254	+0.119	30
51	5617.500	5617.445	+0.055	25
110	5845.000	5845.120	—0.120	9
150	5999.437	5999.476	—0.039	9
201	6196.185	6196.280	—0.095	5
299	6574.555	6574.428	+0.127	18

The following elements (Elements II.) were derived from the foregoing provisional mean maxima :—

$$\text{Maximum} = \text{J.D. } 2415420.641 + 3^d.8589 \text{ E}$$

and the computed times of maximum according to these elements and the residuals O—C are stated in columns 3 and 4 of the above table.

Table IV. contains the observations arranged in order of the time by which they follow the last preceding maximum computed by means of the above elements (II.). The first column gives the number of the observation in Table II., the second the interval following maximum, and the third the observed magnitude. In two or three cases, where several observations were made on the same night, mean values are given in this table. Observations followed by : may possibly have been affected by cloud. Those followed by :: were certainly so affected.

TABLE IV.

Observations in Order of Distance from Maximum.

No. of Obs.	Dist. from Max. d	Mag.	No. of Obs.	Dist. from Max. d	Mag.	No. of Obs.	Dist. from Max. d	Mag.
74	0.03	8.44	91	1.26	9.22	126	2.39	9.65 :
103	0.03	8.65	54	1.31	8.99	34	2.42	9.50
96	0.12	8.78	93	1.33	9.31	84	2.42	[9.19]
97	0.16	8.78	107	1.39	9.12	10	2.44	9.69
109	0.20	8.96	125	1.39	9.31	58	2.45	9.53
90	0.24	8.78	33	1.45	9.26	98	2.51	9.45
121	0.25	8.70	57	1.45	[9.65::]	13	2.53	9.55
80	0.26	8.54	12	1.51	[9.44::]	42	2.58	9.53
111	0.29	9.00	129	1.51	9.33	99	2.58	9.59
29	0.32	8.66	64	1.57	[9.62]	61	2.59	9.53
110	0.33	9.01	41	1.58	[9.29::]	21	2.62	9.67 :
124	0.38	8.78 :	60	1.58	9.26	102	2.71	9.54
83	0.44	8.65 :	101	1.60	9.37	67	2.73	9.61
4	0.45	8.78	19	1.65	9.27	68	2.75	9.64
32	0.45	8.78	66	1.67	9.28	51	2.84	9.65
39	0.56	8.71	50	1.69	9.11	72	2.89	9.52 :
59	0.58	8.60	20	1.71	9.32	117	2.97	9.58
17	0.61	8.77	5	1.72	9.38	73	3.00	9.50
40	0.64	8.86	23	1.75	9.27	95	3.00	9.58
86 } 10 } 88 }	0.64	8.73	116	1.82	9.37	78	3.04	9.57
			24	1.83	9.47	2	3.05	9.69
18	0.69	8.77 :	70	1.87	9.41	113	3.08	9.58
106	0.70	8.88	71	1.99	9.56	108	3.10	9.58
63	0.73	[8.81::]	53	2.00	9.53	119	3.12	9.70
22	0.75	8.93 :	1	2.01	9.40	89	3.13	9.45
25	0.89	9.17 :	105	2.03	9.49	28	3.14	9.65
52	0.98	8.90	77	2.07	9.49	79	3.15	9.67
104	0.99	9.07	118	2.11	9.53	120	3.18	9.52
75	1.03	9.19	8	2.14	[9.77 :]	9	3.24	9.55
112	1.06	9.07	92	2.16	9.52 :	82	3.29	[8.96 :]
7	1.12	8.97 :	81	2.19	9.53	31	3.30	[9.53::]
76	1.15	9.31	27	2.20	9.65	56	3.30	9.59
26	1.17	9.02	123	2.25	9.62 :	11	3.35	9.52
3	1.18	9.17 :	55	2.30	9.45	127	3.36	9.70
122	1.25	9.25	30	2.31	9.65	85	3.40	9.52

No. of Obs.	Dist. from Max. d	Mag.	No. of Obs.	Dist. from Max. d	Mag.	No. of Obs.	Dist. from Max. d	Mag.
128	3'43	9'53 :	115	3'51	9'52	43	3'60	8'89
35 } to }	3'44	9'41	100	3'51	8'91	to }		
38 }			14 } to }	3'54	9'00	49 }		
114	3'47	9'65	16 }			6	3'75	8'80
65	3'48	9'13	62	3'59	9'47	69	3'76	8'71
						94	3'83	8'78

Table V. is Table IV. condensed, the observations having been formed into groups of four* and the means taken. In forming these means the observations bracketed in Table IV. have been rejected. Nearly all of them are marked with either : or ::, indicating that they were respectively possibly or certainly affected by cloud. A few discordant observations not so marked are considered later on.

TABLE V.

Mean Observations.

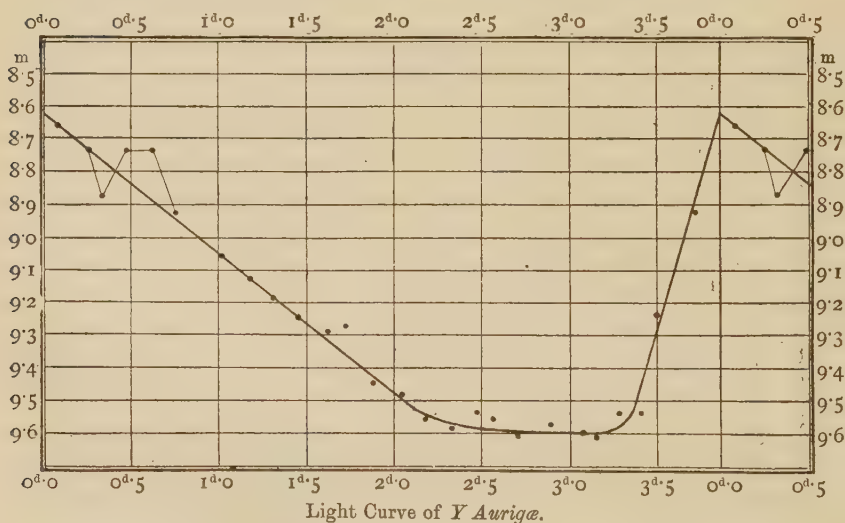
No. of Group.	Dist. from Max. d	Mag. from Curve.	Mag. from Obs.	C—O. mag.	No. of Group.	Dist. from Max. d	Mag. from Curve.	Mag. from Obs.	C—O. mag.
1	0'08	8'66	8'66	'00	14	2'03	9'50	9'48	+ '02
2	0'24	8'74	8'74	'00	15	2'17	9'54	9'56	— '02
3	0'32	8'77	8'87	— '10	16	2'31	9'57	9'59	— '02
4	0'48	8'83	8'74	+ '09	17	2'45	9'58	9'54	+ '04
5	0'62	8'89	8'74	+ '15	18	2'57	9'59	9'56	+ '03
6	0'75	8'94	8'93	+ '01	19	2'71	9'59	9'61	— '02
7	1'01	9'06	9'06	'00	20	2'90	9'60	9'57	+ '03
8	1'16	9'13	9'13	'00	21	3'07	9'60	9'60	'00
9	1'29	9'19	9'19	'00	22	3'13	9'60	9'62	— '02
10	1'43	9'25	9'25	'00	23	3'27	9'57	9'54	+ '03
11	1'62	9'32	9'29	+ '03	24	3'40	9'44	9'54	— '10
12	1'72	9'36	9'27	+ '09	25	3'50	9'27	9'24	+ '03
13	1'88	9'43	9'45	— '02	26	3'71	8'89	8'93	— '04

The above observations having been plotted in diagram form, the mean light curve (B) reproduced in the adjoining figure was drawn. The magnitudes according to this curve are given in the above table next to the observed mean magnitudes, together with the differences C—O. It will be seen from the smallness of these differences in general,† and it will also appear on inspection of

* In the last three cases there are five observations to a group.

† The average difference C—O is ± 0.03 mag.

the diagram, that the observations are satisfactorily represented by the mean light curve excepting at about $0^d.5$ after maximum. The observations in general appear to be represented quite as well by a straight line as by anything else in the nature of a smooth curve between maximum and 2.2 days after that epoch, and I am inclined to think that this is the true form, and that the discordances should be ascribed to errors of observation. As a matter of fact, the comparisons with the comparison star *f* have not always been altogether satisfactory, and an additional comparison star about midway in brightness between this star and *b* is much wanted. However, at least twice as many observations as those included in the present discussion are needed in order to derive a perfectly satisfactory light curve, and it is possible that the discordant observations at about half a day after maximum are not wholly due to errors of observation, but may indicate the existence of a hump or secondary maximum. This must be left to future observation to decide.



The separate observations composing the mean values in the last table are in pretty satisfactory agreement generally, but there are considerable discordances in the case of the last two groups. The star was at this time at the most rapid part of its rise, and a very slight acceleration or retardation in the actual time of maximum consequently here produces a large effect on the brightness of the star. Similar differences are found to occur in the case of other short- (or very short-) period variables, and they are certainly due, in some cases at any rate, to the maximum really occurring a little earlier or later than it would normally do.

With regard to the rejected observations, all of these are marked as being either certainly or possibly affected by cloud, with two exceptions. These are observations 64 and 84 of Tables II. and IV. A re-examination of the original records fails to indicate any reason why these two observations should be discordant. The differences from the mean light curve are respectively $-0^m.39$ and $+0^m.32$.

As the mean light curve B represented in the diagram differs to some extent from the provisional curve A, the times of the mean maxima have been determined again by means of the former curve. The observations were also plotted afresh on squared paper as reduced by the Elements II. The dates and times of these more definitive mean maxima as thus redetermined are stated in Table VI.

TABLE VI.

Definitive Mean Maxima.

E.	Obs. Mean Maximum.	Computed Maximum (III.).	O—C. d	No. of Obs.
9	J.D. 2415455.367	5455.371	$-0^o.004$	33
24	5513.343	5513.256	$+0^o.087$	30
51	5617.417	5617.449	$-0^o.032$	25
110	5845.130	5845.130	$0^o.000$	9
150	5999.407	5999.490	$-0^o.083$	9
201	6196.285	6196.299	$-0^o.014$	5
299	6574.525	6574.481	$+0^o.044$	18

And from the above revised times of maximum the following final elements (Elements III.) have been obtained by the method of least squares :—

$$\text{Maximum} = \text{J.D. } 2415420.64 + 3^d.8590 \text{ E.}$$

The computed times of maximum according to these elements are given in column 3 of Table VI., and the differences O—C in column 4. These differences are a little suggestive of the existence of a third term.

In the *A.N.* 3744 Dr. E. Hartwig published two observations of this star, made on 1901 July 25 and 26 respectively. The magnitude is given as 9.2 on both nights. The former of these observations is $1^d.09$ and the latter $2^d.09$, after the computed time of maximum, and the differences from the mean light curve B are $+0^m.10$ and $-0^m.31$ respectively. The star must have been low and in an unfavourable position for observation at the times of these observations, so that the differences are not greater than what might be expected under the circumstances.

TABLE VII.

Concluded Elements of Y Aurigæ.

Period of variation	...	...	...	$3^d 859^s = 3^d 20^h 36^m 58^s$
Epoch of maximum	...	...	...	1901 Feb. 4, $15^h 22^m$ G.M.T. = J.D. 2415420.64
Maximum brightness	...	...	...	8.63 mag.
Minimum	„	...	...	9.60 „
„ to maximum	...	...	...	$0^d 17^h 31^m$ *
Maximum to minimum	...	...	...	$3^d 3^h 6^m$
Ratio of increase to decrease...	...	...	...	0.23

On the Relative Brightness of Stars. By J. E. Gore.

If the parallax of a star is known, its absolute brightness, or the total amount of light emitted by the star in terms of the Sun's brightness taken as unity, can be easily computed. In the following table I have computed the amount of light in terms of the Sun's light emitted by those stars for which a fairly good parallax has been found. To find the stellar magnitude to which the Sun would be reduced if removed to the distance indicated by the star's parallax, I have used the formula given in my paper in the *Monthly Notices*, vol. lxiii. p. 168 (June 1903), namely

$$M = \log \left(\frac{1}{p} \right) + 0.07$$

where p is the parallax, and Sun's stellar magnitude = -26.5 .

In column 4 the Harvard photometric magnitude is given, except for a few faint stars which are not given in the Harvard Catalogues. In column 5 the best determination of the parallax is given, those marked with an asterisk being perhaps the most reliable; in column 6 the spectrum as observed at Harvard is given; in column 7 the Sun's magnitude if removed to the distance of the star, computed from the above formula; and in column 8 the ratio of the star's light to the Sun's light, if both are supposed placed at the same distance from the Earth. In those cases in which the star is a binary, the mass of the system, computed from the best orbit hitherto computed, is given in the column of remarks.

* But nearly the whole amount of the increase takes place in less than twelve hours.

I	2	3	4	5	6	7	8	
Star.	R.A. 1900.	Dec. 1900.	Phot. Mag.	Parallax.	Spec- trum.	Sun's mag. at Star's distance.	Relative Bright- ness.	Remarks.
	h m	° '		"				
Assiopeiæ ...	0 3·8	+ 58 36	2·42	0·10*	F5G	5·07	11·49	
Cambridge 34	0 12·7	+ 43 27	(7·9)	0·30	...	2·68	0·008	Proper motion = 2''·80
Canæ ...	0 14·9	- 65 28	4 34	0·15*	F8G	4·19	0·87	
Cydri ...	0 20·5	- 77 49	2·90	0·134*	G	4·43	4·09	Proper motion = 2''·28
Assiopeiæ ...	0 43·1	+ 57 17	3·64	0·154*	F8G	4·13	1·57	{ Mass = 1·6222 × Sun's mass
Assiopeiæ ...	1 1·2	+ 54 27	5·34	0·11	H	4·84	0·63	Proper motion = 3''·75
Aris ...	1 18·5	+ 88 43	2·12	0·074	F8G	5·72	27·55	
Aridani ...	1 34·0	- 57 44	0·60	0·043	B5A	6·90	331·18	
Arcti ...	1 39·4	- 16 28	3·65	0·31*	G?	2·61	0·38	
Aridani ...	3 15·9	- 43 27	4·30	0·16	G5K	4·05	0·79	
Aridani ...	4 10·7	- 7 49	4·48	0·166*	H?	3·97	0·62	Proper motion = 4''·05
Baran ...	4 30·2	+ 16 18	1·06	0·107*	K5M	4·92	35·00	
Aldebaran Z. V, 243	5 7·7	- 44 59	(8·5)	0·312*	...	2·60	0·004	Proper motion = 8''·70
Aldebaran ...	5 9·3	+ 45 54	0·21	0·081*	G	5·48	128·23	{ Mass = 12·7 × Sun's mass
Aldebaran ...	6 40·8	- 16 35	- 1·58	0·37*	A	2·23	33·42	Mass = 3·5465 × Sun
Aldebaran ...	7 34·1	+ 5 29	0·48	0·325*	F5G	2·51	6·49	Mass = 3·627 × Sun
Aldebaran ...	7 41·9	- 33 59	5·42	0·064	...	6·04	1·75	
Aldebaran Maj. ...	8 54·2	+ 42 11	4·09	0·20	F5G	3·56	0·61	
Aldebaran ...	9 7·7	+ 53 7	(7·5)	0·14	H	4·34	0·054	
Aldebaran ...	10 3	+ 12 27	1·34	0·022	B8A	8·36	642·7	
Aldebaran ...	10 5·3	+ 49 58	6·76	0·18	H	3·79	0·064	
Cambridge 1646	10 21·9	+ 49 21	6·54	0·11	A?	4·86	0·21	
Aldebaran ...	10 57·9	+ 36 38	7·60	0·47	...	1·71	0·004	Proper motion = 4''·75
Aldebaran ...	11 0·5	+ 44 2	(8·5)	0·24	...	3·17	0·007	Proper motion = 4''·40
Cambridge 1830	11 47·2	+ 38 26	6·47	0·15*	A?	4·19	0·122	Proper motion = 7''·05
Aldebaran ...	12 10·0	- 9 43	6·05	0·14	E	4·34	0·207	
Aldebaran ...	12 21	- 62 32	1·05	0·05	B1A	6·57	165·96	
Aldebaran ...	13 56·8	- 59 53	0·86	0·046	B1A	6·75	226·99	
Aldebaran ...	14 11·1	+ 19 42	0·24	0·024	K	8·17	1486	
Aldebaran ...	14 32·8	- 60 25	0·06	0·75* { K5M	G	0·69	1·786	{ Mass = 2·00 × Sun's mass
Aldebaran ...	16 23·3	- 26 13	1·22	0·021	Ma	8·46	787·04	
Aldebaran ...	17 30·2	+ 55 15	4·2	0·32	A?	2·54	0·216	
Aldebaran ...	17 37·0	+ 68 26	(9·0)	0·25	...	3·08	0·004	
Aldebaran ...	18 0·4	+ 2 31	4·07	0·16*	K	4·05	0·98	Mass = 2·939 × Sun

1	2	3	4	5	6	7	8	
Star.	R.A. 1900.	Dec. 1900.	Phot. Mag.	Parallax.	Spec- trum.	Sun's mag. at Star's distance.	Relative Bright- ness.	Remarks.
α Lyrae ...	18 ^h 33 ^m 6	+ 38° 41'	0.14	0.082	A	5.50	139.32	
α Aquilæ ...	19 45.9	+ 8 36	0.89	0.231*	A5F	3.25	87.91	
61 Cygni ...	21 2.1	+ 38 13	4.96	0.39*	H	2.11	0.07	Proper motion = 5"
δ Equulei ...	21 9.6	+ 9 36	4.61	0.071*	F	5.81	3.02	{ Mass = 1.89 $\times$ Sun Parallax from Sp troscopic measu by Hussey
ϵ Indi ...	21 55.7	- 57 12	4.74	0.28*	K5M	2.83	0.17	
α Gruis ...	22 1.9	- 47 27	2.16	0.015	B5A	9.19	648.63	
Fomalhaut ...	22 52.1	- 30 9	1.30	0.130	A3F	4.50	18.88	
Lacaille 9352 ...	22 59.4	- 36 26	7.1	0.29	...	2.78	0.018	Proper motion = 7"

On the Terms of Long Period in the Complete Expression for the Moon's Longitude. By E. Nevill.

The appearance of Mr. Cowell's Note in the Supplementary Number of the *Monthly Notices* (vol. lxiv. p. 838), on a term of long period in the Moon's longitude indicated by the observed errors in the tabular place of the Moon, suggests the desirability of my forwarding for publication the first few terms of the expression I have deduced for the correction of Hansen's tabular longitudes, which includes a number of terms of long period whose values I have determined.

This portion of the complete expression is

$$\begin{aligned}
 \Delta L = & + \{ 0.56 + 0.44j \} \\
 & + \{ 30.60 + 0.22j \} (T - 1800.50) \\
 & + \{ 4.75 \quad \quad \quad \} (T - 1800.50)^2 \\
 & + 1.15j \sin \{ 4E - 8M + 3J + 79^\circ \} \\
 & + 4.11. \sin \{ 3V - 7E + 4M + 327 \} \\
 & + 15.34. \sin \{ 18V - 16E - a + 35.3 \}^* \\
 & - 14.40. \sin \{ 18V - 16E - a + 31.7 \} \\
 & + 21.47. \sin \{ 8V - 13E + 274.2 \}^* \\
 & - 7.97. \sin \{ 8V - 13E + 227.7 \}
 \end{aligned}$$

$$\begin{aligned}
\Delta L = & - 0''.51 \cdot \sin \{2.18(Y-1850.50) + 130^\circ\} \\
& - 1''.01 \cdot \sin \{2.80(Y-1850.50) + 240^\circ\} \\
& - 1''.83 \cdot \sin \{4.10(Y-1850.50) + 200^\circ\} \\
& + 0''.24 \cdot \sin \{5.03(Y-1850.50) + 46^\circ\} \\
& + 1''.29 \cdot \sin \{6.40(Y-1850.50) + 264^\circ\} \\
& + 0''.55 \cdot \sin \{7.58(Y-1850.50) + 153^\circ\} \\
& + 0''.92 \cdot \sin \{9.69(Y-1850.50) + 185^\circ\}
\end{aligned}$$

+ terms of shorter period than thirty-five years.

The notation explains itself, and the correction is to be subtracted from the tabular longitude. The two terms marked with an asterisk are Hansen's two faulty *Venus* terms, which have to be removed from the tables.

If the observations considered are restricted to those made at Greenwich since 1750, a mere inspection of the outstanding errors shows that they can be represented roughly by an inequality having a period of from seventy to ninety years, and reaching its maximum about 1825.

In my earliest researches of twenty years ago I employed the term

$$+ 2''.80 \sin \{4^\circ.5[Y-1807.52]\}$$

because such a term does exist, and its coefficient had not then been determined. But experience showed that this term did not represent the observations with sufficient closeness, and failed to represent the observations made prior to 1750; so it was replaced by the two terms

$$\begin{aligned}
& + 1''.95 \sin \{2.8[Y-1764.0]\} \\
& + 2''.69 \sin \{4.1[Y-1802.5]\}
\end{aligned}$$

Both represent arguments of actually existing terms whose values had not then been calculated.

Later the development of my theoretical work enabled me to extend the results to those given in the preceding expression, when the last seven inequalities which are expressed in terms of their annual increments do not depend on arbitrary arguments, but have had their annual motion and epoch fixed within narrow limits by their theoretical values.

But if purely empirical terms be employed with arguments of arbitrarily assigned motion and epoch, it is possible to represent the observed values by a thousand different combinations of two or three such terms. But this cannot be legitimate, and any term chosen to represent the outstanding tabular errors must be one that is theoretically possible, and so one whose epoch can be calculated from theory with some degree of approximation.

In addition to the two terms mentioned by Mr. Cowell as

having arguments of the same period as that of the term he has deduced from the observation there are a number of others, such as the three depending on the arguments

$$\begin{aligned} D & - 15V + 12E \\ 2D - 4z + 42V - 40E \\ \omega + 2E - 4M \end{aligned}$$

The values of the coefficients of these terms as calculated by M. Radau's formulæ will be found to be much smaller than is required by Mr. Cowell; but this will be the case with the coefficient of every inequality of this period when so calculated. For the equation employed by M. Radau will not yield a coefficient exceeding a second of arc to any possible term of this period, unless values be assigned to Δb^i or ∇b^i respectively which are far beyond the maximum values of these functions.

Natal Observatory: 1904 December 9.

The Longitude of the Moon's Perigee. By P. H. Cowell.

In this paper I conclude my discussion of the Moon's observed longitude. I discuss the coefficients of $\sin g$ and $\cos g$, and I give values of the Moon's mean longitude and mean longitude of perigee.

To the values of the coefficients of $\sin g$, $\cos g$, derived from columns 53^s2, 53^c2 in Tables VIII., IX., *Monthly Notices*, vol. lxv. December, I have applied the following corrections, based upon the analyses exhibited in Table X. of that paper:

To the coefficients of $\sin g$ —

(i.) $-0''.7 \cos(\omega - \omega') + 0''.2 - 0''.7 \sin(2D - 2g + 3V - 3E)$
for the Airy period;

(iii.) $-0''.7 \cos(\omega - \omega') - 0''.2$

for the Hansen period.

To the coefficients of $\cos g$ —

(i.) $-0''.3 \sin(\omega - \omega') - 0''.7 \cos(2D - 2g + 3V - 3E)$
 $- 1''.0 \sin(-8) + 2e \Delta g$
for the Airy period;

(iii.) $-0''.7 \sin(\omega - \omega')$

for the Hansen period.

Δg is the sum of (i.) the long-period *Venus* term; (ii.) Newcomb's empirical term with the same argument; (iii.) $-11''$.

The values of the corrected coefficients are exhibited in Table I.; the third columns refer to the Hansen period; periods 86-89 are common both to Airy and Hansen, and the results are enclosed in brackets, to call attention to this. In later stages of this paper the values taken for periods 86-89 are the means of the two sets of values here given, and the values for the 133 periods are then treated as continuous.

TABLE I.

Coefficients of $\sin g$, $\cos g$, corrected. Unit 0".1.

Period.	$\sin g$			$\cos g$		
	0 +	48 +	85 +	0 +	48 +	85 +
+ 1	- 2	0	(-3)	+ 4	-10	(-10)
2	-10	0	(-5)	- 3	- 8	(- 5)
3	-10	0	(0)	+ 2	-14	(-18)
4	+ 4	+ 6	(+4)	- 3	-15	(-15)
5	+ 4	+ 9	+5	+ 6	+ 1	- 5
6	+ 6	- 5	+1	+ 3	- 6	- 9
7	+ 4	- 6	+2	- 1	- 5	- 1
8	- 4	+ 6	-7	- 1	- 7	0
9	- 4	- 5	-1	+ 6	- 8	- 3
10	- 1	+ 4	-5	+12	+ 7	- 4
11	-17	- 2	-5	+ 1	+ 3	-12
12	+ 7	+ 2	0	-13	- 3	- 5
13	- 7	- 4	+3	-14	+13	- 6
14	+ 1	- 7	+1	-12	- 4	- 5
15	+ 1	- 2	-3	- 6	0	+ 5
16	+ 7	0	-4	+ 4	+ 6	- 3
17	- 5	- 3	-3	0	- 2	- 6
18	- 1	+ 2	-2	-10	+ 4	- 5
19	- 5	- 9	+2	+ 2	- 1	- 5
20	+ 5	- 1	+3	+ 7	+ 1	- 4
21	- 2	- 2	0	+ 6	+ 1	0
22	+ 2	- 2	0	- 6	- 3	+ 3
23	- 1	+ 1	+2	- 9	+11	+ 1
24	+ 6	- 2	+1	- 6	+ 3	- 7
25	+ 9	- 9	-1	- 6	-10	- 3

Period.	$\sin g$			$\cos g$		
	0 +	48 +	85 +	0 +	48 +	85 +
+ 26	+ 14	+ 7	+ 2	+ 8	+ 1	+ 5
27	- 6	+ 12	+ 6	+ 26	+ 1	+ 2
28	- 1	+ 19	+ 1	- 3	+ 4	+ 5
29	- 2	+ 4	+ 4	+ 4	+ 11	+ 8
30	+ 3	0	+ 5	- 3	0	+ 8
31	- 1	+ 2	- 7	+ 1	+ 2	+ 7
32	- 2	0	- 3	- 17	- 9	0
33	- 2	+ 2	+ 1	+ 6	+ 7	+ 5
34	+ 17	- 3	- 5	- 16	+ 9	- 9
35	- 2	- 6	+ 2	- 9	- 5	- 11
36	- 4	- 1	+ 6	- 1	0	0
37	- 9	- 6	+ 4	- 7	+ 3	- 1
38	+ 4	(- 4)	- 1	+ 4	(- 9)	- 3
39	+ 9	(- 6)	+ 3	+ 1	(- 11)	0
40	- 5	(- 6)	- 3	0	(- 15)	+ 6
41	- 11	(+ 5)	- 4	- 12	(- 13)	+ 4
42	- 7		- 1	- 1		+ 4
43	+ 4		- 3	- 2		+ 1
44	- 2		+ 3	- 11		+ 1
45	+ 8		- 5	- 9		+ 4
46	+ 12		- 3	- 5		+ 1
47	+ 11		- 2	+ 15		+ 3
48	- 3		- 6	+ 2		- 7

It will be noticed that the coefficients of $\sin g$ for the Hansen period are an extremely satisfactory set of numbers, and that they compare favourably with most of the columns of Table VIII., *Monthly Notices*, vol. lxx. December. The values of $\cos g$ are not so accordant. The conclusion, therefore, is that the outstanding corrections required fall into pairs of the form $\sin (g + \phi) - \sin (g - \phi)$ coalescing into $\sin \phi \cos g$; and that the coefficients of $\sin g$ conceal no periodic term with a coefficient greater than 0".3.

To analyse the numbers of Table I. I made use of the methods developed in my paper, *Monthly Notices*, vol. lxx. November. I took sums for 17 consecutive periods (for periods 7 and 127 I took sums for 13 consecutive periods only and then multiplied by $17 \div 13$) and added and subtracted quantities equidistant from period 67.

I thus obtained, in units of $1'' \div 340$, quantities for even and odd function analysis, which I exhibit in Table II.

TABLE II.

Coefficients of $\sin g$ and $\cos g$ arranged for Even and Odd Function Analysis.
Unit $1'' \div 340$.

rot.	$\sin g$		$\cos g$	
	Even.	Odd.	Even.	Odd.
0	-38	...	+ 42	...
1	-11	+45	+ 42	+16
2	- 1	+27	- 8	+54
3	+14	+ 6	-29	+49
4	+57	-17	-111	+33
5	- 6	-46	-142	+16
6	-23	-33	-167	-33
7	- 3	-43	-160	-34
8	-33	- 3	- 89	-31
9	+23	-41	- 78	-24
10	+46	-18	- 16	+14
11	+32	-14	+ 20	+ 2
12	+25	+ 9	- 18	+38
13	-10	+30	- 4	+64
14	-31	+ 9	- 7	+41
15	-55	+23	+16	+18

Although there is the appearance of a run in Table II. in the coefficients of $\sin g$, the quantities involved are exceedingly small (about $0''.1$).

I have calculated the quantities x defined in my paper, lxxv. November, with the following results :

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
$\sin g$ } unit $1'' \div 340$ }	-12		+69		-7		-33		+34
		-25		-68		-26		+17	
$\cos g$ } unit $1'' \div 340$ }	+53		+37		+3		-94		+58
		+82		+102		-171		+154	
Δg } unit $1'' \div 20$ }	+28		+20		+2		-50		+31
		+43		+54		-91		+82	

A discordance between tables and observation in the coefficient of $\cos g$ implies a discordance nine times as great in the value of g . The third set of values given above has therefore been derived from the second by dividing by 17, to reduce the unit to $0''.05$, and multiplying by 9.

At this point it is convenient to set down the tabular values of the mean longitude &c. that have been employed.

Airy's arguments are taken from Damoiseau's tables, 1824. I give them here reduced from Damoiseau's epoch, which Airy takes as 1801 January 0^m 5—9^m 21^s·5 G.M.T. to Hansen's epoch of 1800 January 0·0 G.M.T. Next I copy Hansen's arguments g , g' , ω , ω' , $-\Omega$, and calculate from them L , L' .

Arg.	At 1800 Jan. 0·0 G.M.T.	Coefficient of T.	Coefficients of	
		rev.	T^2 .	T^3 .
Damoiseau.	° ' "	° ' "		
L	335 43 28·70	1336+307 52 41·39	+ 10·723	+ 0·019361
g	110 19 47·63	1325+198 49 53·99	+ 50·418	+ 0·091035
$-\Omega$	326 43 49·40	5+134 9 57·63	- 6·563	- 0·011850
L'	279 54 40·45	100+ ... 45 53·00	...	...
g'	... 25 17·39	100- ... 57 17·00	...	...
Hansen.				
g	110 19 33·64	1325+198 50 37·15	+ 49·435	+ 0·050073
g'	... 24 28·22	100- ... 56 32·18	- 0·561	...
ω	192 7 21·91	16+243 12 2·07	- 44·323	- 0·043759
ω'	246 13 50·28	5+135 51 38·09	- 6·518	- 0·007159
$-\Omega$	326 43 28·85	5+134 8 59·61	- 8·189	- 0·007159
L	335 43 26·70	1336+307 53 39·61	+ 13·301	+ 0·013473
L'	279 54 49·65	100+ ... 46 6·30	+ 1·110	...
Hansen— Damoiseau.				
L	... - 2·00	+ 58·22	+ 2·578	- 0·005888
g	... - 13·99	+ 43·16	- 0·983	- 0·040962
L'	... + 9·20	+ 13·30	+ 1·110	...
g'	... - 49·17	+ 44·82	- 0·561	...
$-\Omega$	... - 20·55	- 58·02	- 1·626	+ 0·004691

To Hansen's tabular L Professor Newcomb has applied the correction in the *Nautical Almanac* since 1883

$$-1''\cdot14 - 29''\cdot17 T - 3''\cdot76 T^2$$

I have applied the same correction to the individual tabular places from 1847 to 1882.

In my paper in *Monthly Notices*, vol. lxv. November, I have applied to the means for each period of analysis 1-89

$$-3''\cdot14 + 28\cdot63 T - 1''\cdot1828 T^2.$$

This was intended to secure a uniform system of tabular places for the discussion of that paper; but it will be seen that owing to an error a correction $+0''\cdot42 T$ is still required by Airy's tabular places. As this correction only varies from $-0''\cdot2$ to $+0''\cdot2$ it is unimportant.

On certain assumptions as to the nature of the long-period empirical term required it appears from the opening paragraphs of the paper quoted that a correction

$$-4''\cdot3 \text{ or} \\ -4''\cdot3 + 5''\cdot2 - 3''\cdot0 T - 4''\cdot4 T^2$$

is still required. Probably it is best to assume such a period for the empirical term as will make the secular acceleration equal to its theoretical value. The correction will then be about

$$-4''\cdot3 + \frac{1}{2}\{ + 5''\cdot2 - 3''\cdot0 T - 4''\cdot4 T^2 \}$$

to which we must add $+0''\cdot8$ if the formula is to represent the observations after 1902.0, which are now reduced to the epoch of Newcomb's catalogue. The mean longitude is therefore

$$335^\circ 43' 25'' + (1336^{\text{rev.}} + 307^\circ 53' 9'')T + 7''\cdot3 T^2$$

subject to uncertainty of the order

$$\pm 5'' \pm 3'' T$$

until theory has given a more precise value to the empirical terms.

Coming now to the mean anomaly, Professor Newcomb has applied the correction

$$-1''\cdot14 - 29''\cdot17 T - 3''\cdot76 T^2$$

in the *Nautical Almanac* since 1883. I have applied the same correction to the individual tabular places from 1847 to 1882. I have also applied (see *Monthly Notices*, vol. lxiv. p. 85)

$$+4''\cdot63 - 13''\cdot99 T + 4''\cdot743 T^2$$

to all tabular values of g based on Hansen. It will be seen that the value of g for the Hansen period has now been reduced to $-10''\cdot5$ (a constant) in excess of that used by Airy.

Owing to a numerical error, in the first table of this paper I have applied $-11''\cdot0$ to Airy's g ; there is therefore a want of continuity of $0''\cdot5$ in g , or Airy's tabular places require a further correction of $+0''\cdot05 \cos g$. This is insensible.

In Table I. g also contains a long-period *Venus* term and Newcomb's empirical term of the same period. In my paper (*Monthly Notices*, vol. lxv. November) I came to the conclusion that Newcomb's empirical term had too short a period. Removing it therefore by applying to the x 's (see *Monthly Notices*, vol. lxv. p. 51)

$$\begin{array}{cccccc} +53\cdot2 & -69 & +2 & -1 & -2 \\ +316 & -19 & 0 & 0 & 0 \end{array}$$

we obtain for the x 's of Δg

$$\begin{array}{ccccc} +560 & -49 & +4 & -51 & +29 \\ +359 & +35 & -91 & +82 & \end{array}$$

If the value $+359$ of x_2 be attributed to a secular term in g we should obtain a concluded value entirely at variance with theory. Analogy with the long-period *Venus* term leads us to expect that the cause that produces the empirical term in the longitude will also produce the same term in the mean anomaly. I therefore apply the x 's of my long-period empirical term (see *Monthly Notices*, vol. lxx. p. 51), which are :

$$\begin{array}{ccccc} -464 & +32 & 0 & 0 & 0 \\ -237 & +8 & 0 & 0 & \end{array}$$

obtaining for the corrected x 's of g

$$\begin{array}{ccccc} +96 & -17 & +4 & -51 & +29 \\ +122 & +43 & -91 & +82 & \end{array}$$

A better way of treating the subject is to form the x 's of $L-g = \varpi$, the longitude of perigee. These are :

$$\begin{array}{ccccc} -86 & +20 & -35 & +46 & -13 \\ -86 & +39 & +21 & -68 & \end{array}$$

It will be noticed that the effect of this last operation is identical with the effect of changing the sign of the row of x 's immediately above, after adding in the x 's of the two empirical terms (see *Monthly Notices*, vol. lxx. November) of 66 and 42 years' period.

As the x 's for ϖ are smaller than the preceding set, to that very slight extent there is evidence from the observations for my two smaller empirical terms being accompanied by the terms formed by multiplying them by $2e \cos g$.

In the x 's for ϖ I am obliged to dismiss as accidental all but $x_1 = -86$ and $x_2 = -86$. I can to some extent justify this by pointing out that the x 's for $\sin g$ are probably accidental. For $\sin g$, $x_3 = +69$. Multiplying by the ratio $9 : 17$ we get $+37$. I am now proposing to treat -68 the x_3 for ϖ as accidental. It is larger than I could wish, but I see no alternative. It must be admitted therefore that x_1 and x_2 are subject to possible errors ± 80 .

The tabular longitude of perigee used in this discussion is Hansen's $\omega + \delta$ increased by

$$-4''\cdot63 + 13''\cdot99 T - 4''\cdot74 T^2.$$

This requires correction by

$$+86 t_1 + 86 t_2$$

or by Constant $+2''.7 T + 7''.4 T^2$, measuring T from 1826

or by Constant $-1''.0 T + 7''.4 T^2$, measuring T from 1800.

To find the constant, the mean of the quantities in Table III. (*Monthly Notices*, vol. lxv. p. 39) is $-1''.35$; the mean of the quantities in the last column of the first table of this paper is $-0''.185$, corresponding to $\Delta g = -1''.67$; the mean value of the correction to ϖ must therefore be $-0''.32$; the constant is therefore $-3''.7$, to which must be added $+0''.8$ for the epoch of Newcomb's catalogue. The concluded value therefore exceeds Hansen's by

$$-7''.5 + 13''.0 T + 2''.7 T^2$$

or

$$\varpi = 225^\circ 23' 46'' + (11^{\text{rev.}} + 109^\circ 3' 15'') T - 33'' T^2$$

with possible errors of $3''$ in the mean motion for 1826 and $7''$ in the secular term, and perhaps $2''$ in the value for 1875.

Reducing the motion of the perigee to 1850 and subtracting $5024''$ for precession, I obtain for the observed sidereal motion of the perigee

$$14643538'' \text{ with a possible error of } \pm 6''.$$

Professor Brown (*Monthly Notices*, vol. lxiv. p. 532) quotes $14643523''$ as the observed value, which agrees more closely with his theoretical values than the observed value found by me.

My phrase "possible error" may be approximately taken to mean three times the probable error. I am assuming 20, 35, 46, 13, 39, 21, 68 to be accidental errors, and I am taking 80 as the measure of the "possible error."

As this paper concludes my discussion of the longitude I here give a summary of the results obtained.

1. Certain constants have been measured.

2. Table VIII. (*Monthly Notices*, vol. lxv. December) shows a close agreement between theory and observations as regards short-period terms. Probably there is no unknown term with coefficient $0''.4$ or more having a mean motion differing by less than $0^\circ.4$ a day from the mean motion of any one of the 40 auxiliary angles of that paper.

3. An apparent exception, depending on the argument $g + \omega - \omega'$ or possibly $g + \varpi$ is probably to be attributed to errors of north polar distance.

4. The coefficient ($6''.6$) of $\sin \Omega$, the principal figure of Earth term, is decidedly smaller than the latest theoretical value. ($7''.7$ G. W. Hill).

5. Certain empirical terms have been obtained (*Monthly Notices*, vol. lxv. November).

The Final Values of the Coefficients in the New Lunar Theory.
By Ernest W. Brown, Sc.D., F.R.S.

1. As has been stated on previous occasions the problem under consideration, and now completed, is that of Delaunay's theory, with the additional terms introduced by replacing a/a' by $a(E-M)/a'(E+M)$. In earlier papers* I have given a general account of the methods used and of the means taken to secure accuracy, with some indication of the extent to which the results conform with those deduced from observation. The main object of the present communication is to give the complete numerical values of the coefficients of all periodic terms in longitude and latitude which are as great as $0''.01$, and in parallax those which are as great as $0''.001$. Every coefficient has been taken to at least one more place in the computations.

A secondary object is to compare the results with those of Hansen, so as to show explicitly the extent of the agreement between the two theories. The results of Delaunay may be used as a check where differences from those of Hansen occur; but slow convergence makes so many of Delaunay's coefficients doubtful that it does not seem useful to give them here. They can, if necessary, be directly obtained from Newcomb's "Transformation of Hansen's Theory,"† the values which he there finds for the latter being inserted in the tables below. The general result of this comparison, stated briefly, is that where sensible differences occur between my results and those of Hansen my values are confirmed by Delaunay when allowance is made for the slow convergence of his series.

The mean motions of the perigee and node are not given here. Several comparisons with the results of Delaunay have been made on previous occasions, and the final numbers are fully set out and discussed in a paper "On the Degree of Accuracy of the New Lunar Theory and on the Final Values of the Mean Motions of the Perigee and Node."‡

* In particular those in the *Monthly Notices* for April 1904 and December 1904.

† *Washington Astr. Papers*, vol. i. pt. (ii.)

‡ *Monthly Notices*, 1904 April, pp. 524-534. I am much indebted to Professor H. L. Rice of the Naval Observatory, Washington, for pointing out an error in the paragraph numbered 5 in this paper. The expressions

$$-\frac{3\delta a'}{a'} \cdot \frac{0''.70}{328243}, + \frac{3\delta a'}{a'} \cdot \frac{0''.20}{328243}$$

should be replaced by

$$-328243 \frac{3\delta a'}{a'} 0''.70, + 328243 \frac{3\delta a'}{a'} 0''.20,$$

and all the numbers in the second and third columns of the table headed "Indirect Planetary Action" (p. 529) should be multiplied by 1.077. Fortunately the changes in the final results are almost insensible: $-0''.01$ is to be added to the calculated results for the perigee given on pp. 525, 532, and $+0''.01$ to the calculated results for the node on the same pages.

2. *The Constants of the Theory.*—The numerical values of the constants used in reducing the theory to numbers are as follows :

$$\text{MOON (1850.0).}$$

$$n = 17\ 325\ 594''\cdot 06$$

$$e = \cdot 054\ 900\ 56$$

$$\gamma = \cdot 044\ 887\ 16$$

$$\frac{1}{a} = 3412''\cdot 596$$

$$\text{SUN (1850.0).}$$

$$n' = 1\ 295\ 977''\cdot 415$$

$$e' = \cdot 016\ 771\ 91$$

$$\gamma' = 0$$

$$\frac{1}{a'} = 8''\cdot 7800$$

$$\frac{E}{M} = 81\cdot 500.$$

The definitions of these constants are those adopted by Delaunay.

The object here being the comparison of the two sets of *theoretical* coefficients and not the comparison of either with the observed values, Hansen's coefficients are given directly from Newcomb's results, referred to above, and the changes which they require when Hansen's constants are altered to the values just given are shown separately. The comparison of the values for e , γ , a is seen directly from the coefficients of $\sin l$ and $\sin F$ and $\cos 0$ in the longitude, latitude, and parallax respectively. Hansen's $e' = \cdot 016\ 792\ 28$, and his $1/a' = 8''\cdot 848$. His values for n , n' are the same as mine within the limits of accuracy of the comparison.*

3. *Explanation of the Tables Below.*—The *first* column gives the principal characteristic (C) of each set of terms. In the *second*, *third*, *fourth*, and *fifth* columns are the multiples of l , l' , F , D (Delaunay's notation), which enter into the arguments ; the characteristic and multiples of l , l' , F being the same for each set of terms (that is, for those terms whose arguments differ only by multiples of $2D$), they are only set down for the first term of each set. The *sixth* column (headed B) contains my final values. The *seventh* column (headed H) contains Hansen's theoretical values, with his constants. The *eighth* column gives the reduction (R) necessary to reduce Hansen's results to my set of constants. The *last* column (B—H—R) shows the real differences between the results of the two theories. The coefficients to which letters are attached are discussed in the following section.

4. *Observations on the Results.*—As has been stated the calculations were constructed so as to include all coefficients in longitude and parallax as great as $0''\cdot 01$, and to neglect all characteristics which did not have at least one coefficient as large as this amount. But since the calculations in each characteristic included all coefficients of $0''\cdot 0005$ and over, there are com-

* A typographical error occurs on p. 526, where Newcomb's coefficient for the principal elliptic inequality is set down as $22659''\cdot 58$; it should, of course, be $22639''\cdot 58$.

paratively few lying between $0''.005$ and $0''.01$ (which are of course entered as equal to $0''.01$) present in Hansen's theory and not in mine. In longitude there are but two, and for one of these, characteristic e^6 and argument $6l$, the elliptic value suffices; the other ($0''.01$) has the characteristic $e^3e'\gamma^2$, and Hansen and Delaunay agree on its value. In latitude there are two, with characteristic $\gamma e^4e'$, to which the previous remark also applies. For the parallax similar remarks may be made with the degree of accuracy $0''.001$ at the start; there are three such coefficients, characteristic e^4e' , each having a coefficient, according to Hansen, of $0''.001$, and they are probably less than $0''.001$ and greater than $0''.0005$. These are, of course, quite unimportant from a practical point of view, and they really only need consideration when the number of them is comparatively large. It is, therefore, of some interest to know the sum of the absolute values of the differences $B-H-R$ in each coordinate to obtain an idea of the maximum differences which tables constructed on the two theories would show. Adding the numbers in the columns $B-H-R$, without regard to sign, we obtain

In longitude	...	...	...	...	3'61
In latitude	...	...	...	...	1'90
In parallax	...	...	...	...	'242

The great majority of these differences are in the coefficients of short-period terms. Also, there are few which exceed $0''.02$ in longitude and latitude and $0''.002$ in parallax.

Certain coefficients require special mention. In that of the *variation*, marked (a) below, there is a difference of $0''.16$. Newcomb's comparison shows a practical agreement between Hansen and Delaunay. In an earlier paper* I stated that Delaunay's value for the part of this coefficient with characteristic e^2 was in error by about $0''.10$, and the correction has been confirmed by M. Andoyer.† This accounts for all but $0''.06$ of the difference between my result and that of Delaunay, and an estimate for neglected terms in his series must be uncertain by at least this amount.

In the *parallactic inequality*, marked (b), there is a difference of $-0''.32$. I have given‡ a comparison between my results and those of Delaunay for the various parts of this coefficient. This comparison showed that $-0''.21$ must be added to the latter to account for neglected terms; an estimate for such terms

* *Monthly Notices*, vol. lvii. (1897), p. 336.

† *Comptes Rendus*, vol. cxxx. (1900), p. 1532.

‡ *Monthly Notices*, April 1904, p. 534. My calculated results only included the terms of characteristics of orders 1, 3. Those whose characteristics were of order 5 were estimated from Delaunay's series to amount to $+0''.02 \pm 0''.02$. Complete calculations for the latter have since shown that these terms give $+0''.024$, so that the results of that paper are correct to the degree of accuracy there adopted.

confirms my value within the limits of possible error of the estimate.

In the coefficients, marked (c), the differences are $0''.23$ in a coefficient $1''.09$, and $0''.21$ in a coefficient $1''.30$. Newcomb estimates $0''.87$ and $1''.39$ as Delaunay's complete values for the respective coefficients, but any estimates must be uncertain by at least the differences between the two sets of results. The periods are long and the coefficients are difficult to determine by any method in which approximation along powers of m is used. Even with the method of this theory the loss of accuracy owing to small divisors is so great that these two coefficients to a certain extent determine the number of places of decimals to be adopted at the outset of the whole work.*

It is not evident why the differences marked (f), which depend mainly on γ , practically disappear if we adopt $18461''.5$ instead of $18463''.3$ for the principal term in latitude of Hansen's theory, other differences being not materially affected. It would almost appear as though a value near the former was really the constant of Hansen's theory, especially as these coefficients are easy to determine accurately.

True Longitude—Mean Longitude. Coefficients of Sines.

C.	I.	V.	F.	D.	B.	H.	R.	B-(H+R).		
I	0	0	0	6	+	0''13	+	'13	...	...
			4	+	13'90	+	13'90	...	...	...
			2	+	2369'90	+	2369'75	-0'01	+	'16 (a)
e	I	0	0	6	+	'02	+	'02	...	...
			4	+	1'98	+	1'98	...	...	...
			2	+	191'95	+	191'95	...	...	...
			0	+	22639'58	+	22640'15	-57	...	...
			-2	-	4586'44	-	4586'56	+1'11	+	'01
			-4	-	38'43	-	38'43	...	...	...
			-6	-	'39	-	'40	...	+	'01
e'	0	I	0	4	-	'29	-	'29	...	...
			2	-	24'45	-	24'45	+0'03	-	'03
			0	-	668'94	-	669'85	+81	+	'10
			-2	-	165'35	-	165'22	+20	-	'03
			-4	-	1'88	-	1'89	...	+	'01
			-6	-	'02	-	'02	...	...	...
a	0	0	0	5		'00	+	'01	...	-0'01
			3	+	'40	+	'41	...	-	'01
			1	-	124'79	-	125'43	+96	-	'32 (b)

* See a paper by the writer, "On the Small Divisors in the Lunar Theory," *Trans. Amer. Math. Soc.* vol. iii. (1902), pp. 159-185. The transformations given in sect. iii. of the paper were unfortunately only worked out after most of the calculations had been completed, so that they have not been used for computation.

True Longitude—Mean Longitude. Coefficients of Sines.

C.	L.	V.	F.	D.		B.		H.		R.		B-(H+R).
e^2	2	0	0	4	+	'21	+	'22	...			-'01
				2	+	14'39	+	14'38	...			+'01
				0	+	769'02	+	769'06	- '04			...
				-2	-	211'66	-	211'71	+'01			+'04
				-4	-	30'77	-	30'78	...			+'01
				-6	-	'57	-	'57	...			...
				-8	-	'01		...	...			-'01
ee'	1	1	0	4	-	'05	-	'05	...			...
				2	-	2'93	-	2'93	...			...
				0	-	109'80	-	109'92	+'14			-'02
				-2	-	206'22	-	206'46	+'26			-'02
				-4	-	4'40	-	4'41	+'01			...
				-6	-	'07	-	'07	...			...
	1-1	0	6	+		'01		...	...			+'01
				4	+	'28	+	'29	...			-'01
				2	+	14'60	+	14'60	- '02			+'02
				0	+	147'88	+	148'02	- '18			+'04
				-2	+	28'51	+	28'56	- '03			-'02
				-4	+	'64	+	'64	...			...
				-6	+	'01	+	'01	...			...
e'^2	0	2	0	2	-	'19	-	'19	...			...
				0	-	7'51	-	7'51	+'02			-'02
				-2	-	8'12	-	8'13	+'02			-'01
				-4	-	'15	-	'15	...			...
γ^2	0	0	2	4	-	'09	-	'09	...			...
				2	-	5'74	-	5'74	...			...
				0	-	411'61	-	411'60	+'08			-'09(f)
				-2	-	55'17	-	55'25	+'01			+'07
				-4	+	'03		...	...			+'03
ea	1	0	0	1	-	8'44	-	8'48	+'07			-'03
				-1	+	18'55	+	18'70	- '14			-'01
				-3	+	3'21	+	3'23	- '02			...
				-5	+	'01	+	'01	...			...
$e'a$	0	1	0	3	+	'15	+	'15	...			...
				1	+	17'99	+	18'09	- '16			+'06
				-1	+	'56	+	'60	...			-'04
				-3	-	'07	-	'06	...			-'01

True Longitude—Mean Longitude. Coefficients of Sines.

C.	L.	V.	F.	D.		B.		H.		R.		B-(H+R).
$e'\gamma^2$	0	1	2	2	+	'07	+	'06	...	...	+	'01
				0	+	'42	+	'42	...	...	...	...
				-2	-	2'16	-	2'15	...	...	-	'01
				-4	-	'01	-	'04	...	...	+	'03
	0	1-2	4			'00	-	'02	...	...	+	'02
				2	-	1'44	-	1'55	...	...	+	'11
				0	+	'08	+	'08	...	...	...	...
				-2	+	'38	+	'38	...	...	...	...
				-4	+	'01	+	'01	...	...	...	...
$e^2\alpha$	2	0	0	1	-	'58	-	'59	...	...	+	'01
				-1	+	1'75	+	1'78	-	'01	-	'02
				-3	+	1'22	+	1'22	-	'01	+	'01
				-5	+	'06	+	'06	...	...	...	...
$ee'\alpha$	1	1	0	3	+	'02	+	'02	...	...	...	...
				1	+	1'27	+	1'27	-	'01	+	'01
				-1	+	'14	+	'17	...	...	-	'03
				-3	+	'23	+	'23	...	...	...	...
	1-1	0	1	-		'12	-	'13	...	...	+	'01
				-1	-	1'09	-	1'33	+	'01	+	'23 (c)
$e'^2\alpha$	0	2	0	1	-	'04	-	'03	...	...	-	'01
				-1	-	'04	-	'04	...	...	...	...
				-3	-	'01	-	'01	...	...	...	...
$\gamma^2\alpha$	0	0	2	1	+	'25	+	'25	...	...	...	...
				-1	+	'58	+	'55	...	...	+	'03
				-3	+	'25	+	'32	...	...	-	'07
e^4	4	0	0	2	+	'07	+	'08	...	...	-	'01
				0	+	1'94	+	1'94	...	...	...	...
				-2	-	'95	-	'95	...	...	...	...
				-4	-	'00	+	'03	...	...	-	'03
				-6	-	'01	-	'01	...	...	...	...
e^3e'	3	1	0	2	-	'03	-	'02	...	...	-	'01
				0	-	'55	-	'55	...	...	...	...
				-2	-	'48	-	'48	...	...	...	...
				-4	-	'10	-	'07	...	...	-	'03
				-6	-	'04	-	'04	...	...	...	...
	3-1	0	2	+		'09	+	'10	...	...	-	'01
				0	+	68	+	'67	...	...	+	'01
				-2	-	'18	-	'18	...	...	...	...
				-4	-	'03	-	'04	...	...	+	'01
				-6	+	'01	...	...	...	...	+	'01

True Longitude—Mean Longitude. Coefficients of Sines.

C.	L.	V.	F.	D.	B.	H.	R.	B-(H+R).
$e^2e'^2$	2	2	0	0	—	" '07	—	'06 ... —'01
			—2	—	'30	—	'28 ...	—'02
			—4	—	'16	—	'16 ...	...
			—6	—	'01	...	...	—'01
	2—2	0	2	+	'06	+	'06 ...	...
			0	+	'20	+	'19 ...	+ '01
			—2	+	'26	+	'23 ...	+ '03
			—4	+	'04	+	'03 ...	+ '01
ee'^3	1	3	0	0	—	'02	—	'02
			—2	—	'25	—	'26 ...	+ '01
			—4	—	'02	—	'02 ...	...
	1—3	0	2	+	'03	+	'03 ...	...
			0	+	'05	+	'05 ...	...
e'^4	0	4	0—2	—	'01	...	...	—'01
$e^2\gamma^2$	2	0	2	2	—	'12	—	'12
			0	—	4'00	—	4'00 ...	...
			—2	+	'56	+	'56 ...	...
			—4	—	'01	...	...	—'01
	2	0—2	4	—	'01	—	'01 ...	...
			2	—	'46	—	'43 ...	—'03
			0	—	1'30	—	1'09 ...	—'21 (c)
			—2	+	'54	+	'54 ...	...
			—4	+	'17	+	'17 ...	...
			—6	+	'01	...	...	+ '01
$ee'\gamma^2$	1	2	2	+	'01	+	'02 ...	—'01
			0	+	'26	+	'27 ...	—'01
			—2	+	'06	+	'06 ...	...
			—4	—	'02	—	'03 ...	+ '01
	1	1—2	2	+	'08	+	'08 ...	...
			0	—	'08	—	'08 ...	...
			—2	+	'43	+	'43 ...	...
			—4	+	'02	+	'02 ...	...
	1—1	2	2	—	'06	—	'08 ...	+ '02
			0	—	'30	—	'30 ...	...
			—2	—	'00	+	'01 ...	—'01
			—4	+	'02	+	'03 ...	—'01
	1—1—2	4	—	—	'01	...	...	—'01
			2	—	'37	—	'40 ...	+ '03
			0	+	'08	+	'09 ...	—'01
			—2	—	'07	—	'07 ...	...

True Longitude—Mean Longitude. Coefficients of Sines.

O.	L.	l.	F.	D.	B.	H.	R.	B-(H+R).
$e'2\gamma^2$	0	2	2-2	-	'07	-	'07	...
	0	2-2	2	-	'03	-	'01	...
		0			'00	...	...	...
		-2	+		'02	+	'02	...
γ^4	0	0	4	2	+	'01	+	'01
		0	+		'42	+	'42	...
		-2	+		'07	+	'08	...
$e3a$	3	0	0	1	-	'04	-	'04
		-1	+		'13	+	'12	...
		-3	+		'05	+	'06	...
		-5	+		'02	...	...	...
$e2e'a$	2	1	0	1	+	'09	+	'09
		-1	+		'01	+	'01	...
		-3	+		'08	+	'08	...
		-5	+		'01	+	'01	...
	2-1	0	1	-	'01	-	'02	...
		-1	-		'35	-	'38	...
		-3	+		'04	+	'04	...
$ee'a_1$	1	2	0	1	-	'01	'00	...
		-1			'00	...	...	...
		-3	+		'01	+	'01	...
	1-2	0-1			'00	-	'01	...
$e\gamma^2a$	1	0	2	1	+	'05	+	'04
		-1	+		'02	+	'01	...
		-3	+		'03	...	...	...
	1	0-2	3	-	'01	-	'01	...
		1	-		'04	-	'08	...
		-1	-		'02	-	'02	...
		-3	-		'01	-	'01	...
$e'\gamma^2a$	0	1	2	1	-	'04	-	'04
		-1	+		'01	...	...	...
		-3	+		'02	...	...	...
	0	1-2	3	+	'01	...	...	...
		1			'00	-	'06	...
$e5$	5	0	0	2	+	'01	...	...
		0	+		'11	+	'11	...
		-2			'07	-	'07	...
		-4			'00	+	'01	...

True Longitude—Mean Longitude. Coefficients of Sines.

O.	I.	V.	F.	D.	B.	H.	R.	B-(H+R).			
e4e'	4	1	0	0	-	'04	-	'04	...	...	
				-2	-	'03	-	'03	...	...	
	4-1	0	2	+	'01	...	...	...	+	'01	
				0	+	'05	+	'05	...	...	
				-2	-	'02	-	'02	...	...	
e3e' ²	3	2	0	-2	-	'02	-	'02	...	...	
				-4	-	'01	...	...	...	- '01	
	3-2	0	2	+	'01	...	...	...	+	'01	
				0	+	'02	+	'01	...	+	'01
				-2	+	'01	+	'01	...	...	
e2e' ³	2	3	0	-2	-	'01	-	'01	...	...	
				-4	-	'01	...	...	...	- '01	
e3γ ²	3	0	2	2	-	'01	-	'01	...	...	
				0	-	'33	-	'33	...	...	
				-2	+	'09	+	'10	...	- '01	
	3	0	-2	2	-	'03	-	'03	...	...	
				0	-	'06	-	'07	...	+	'01
				-2	-	'01	-	'01	...	...	
				-4	+	'01	+	'01	...	...	
e2e'γ ²	2	1	2	0	+	'04	+	'04	...	...	
				-2	+	'03	+	'03	...	...	
	2	1	-2	2	+	'01	+	'01	...	...	
				0	+	'03	-	'07	...	+	'10
				-2	+	'02	+	'02	...	...	
				-4	+	'02	+	'01	...	+	'01
	2-1	2	2	-	'01	-	'01	...	...	...	
				0	-	'05	-	'05	...	...	
				-2	+	'00	+	'01	...	- '01	
	2-1	-2	2	-	'03	-	'03	...	...	...	
				0	-	'02	-	'01	...	- '01	
ee' ² γ ²	1	2	2	-2	-	'00	+	'01	...	- '01	
	1	2	-2	-2	+	'02	+	'02	...	...	
	1-2	2	0	-	'01	...	...	...	...	- '01	
				-2	+	'01	...	...	...	+	'01
	1	-2	-2	2	-	'02	+	'01	...	- '03	
				0	-	'00	...	...	...	...	
				-2	-	'01	...	...	...	- '01	

True Longitude—Mean Longitude. Coefficients of Sines.

C.	L.	V.	F.	D.	B.	H.	R.	B-(H+R).		
$e\gamma^4$	1	0	4	0	+	"09	+	'09	...	...
				-2	+	'01	+	'01	...	...
	1	0	-4	2		'00	+	'03	...	- '03
				0	-	'08	-	'08	...	...
				-2	-	'02	-	'02	...	...
$e4a$	4	0	0	-1	+	'01	+	'01	...	...
$e3e'a$	3	1	0	1	+	'01	+	'01	...	...
	3	-1	0	-1	-	'02	-	'02	...	...
				-3	+	'01	...	...	...	+ '01
$e2\gamma^2a$	2	0	2	1	+	'01	...	...	...	+ '01
	2	0	-2	1		'00	-	'01	...	+ '01
$ee'\gamma^2a$	1	1	2	1	-	'01	-	'01	...	...
	1	1	-2	1		'00	-	'05	...	+ 0'5
$e6$	6	0	0	0	+	'01	+	'01	...	...
$e4\gamma^2$	4	0	2	0	-	'03	-	'03	...	...
				-2	+	'01	+	'01	...	...
	4	0	-2	0	-	'01	...	...	...	- '01
$e2\gamma^4$	2	0	4	0	+	'01	+	'01	...	...
	2	0	-4	0		'00	+	'01	...	- '01
$e3e'\gamma^2$	3	-1	2	0	Not cal.	-	'01	...	...	

Latitude. Coefficients of Sines.

C.	L.	V.	F.	D.	B.	H.	R.	B-(H+R).		
γ	0	0	1	6	+	'02	+	'01	...	+ '01
				4	+	1'19	+	1'19	...	...
				2	+	117'26	+	117'26	- '01	+ '01
				0	+	18461'48	+	18463'25	- 1'77	...
				-2	-	623'66	-	623'70	+ '06	- '02
				-4	-	3'68	-	3'68	...	...
			-6	-	'04	-	'04	...	...	
$\gamma\epsilon$	1	0	1	4	+	'21	+	'21	...	...
				2	+	15'12	+	15'12	...	...
				0	+	1010'18	+	1010'17	- '12	+ '13 (f)
				-2	-	166'58	-	166'60	+ '02	...
				-4	-	6'58	-	6'58	...	...
				-6	-	'10	-	'09	...	- '01
	-1	0	1	6	+	'04	+	'04	...	...
				4	+	3'00	+	3'00	...	...
				2	+	199'49	+	199'48	- '02	+ '03 (f)
				0	-	999'70	-	999'69	+ '12	- '13 (f)
				-2	-	33'36	-	33'37	...	+ '01
				-4	-	'48	-	'47	...	- '01
			-6	-	'01		'00	...	- '01	

Latitude. Coefficients of Sines.

C.	L.	V.	F.	D.	B.	H.	R.	B-(H+R).		
$\gamma e'$	0	1	1	4	-	'02	-	'03	...	+ '01
				2	-	1'27	-	1'28	...	+ '01
				0	-	6'49	-	6'50	+ '01	...
				-2	-	29'69	-	29'74	+ '04	+ '01
				-4	-	'42	-	'41	...	- '01
				-6	-	'01	...	...	...	- '01
	0-1	1	1	4	+	'15	+	'16	...	- '01
				2	+	8'00	+	8'00	- '01	+ '01
				0	+	4'86	+	4'88	- '01	- '01
				-2	+	12'14	+	12'14	- '01	+ '01
				-4	+	'11	+	'10	...	+ '01
γa	0	0	1	3	-	'03	-	'03	...	...
				1	-	5'36	-	5'25	+ '04	- '15
				-1	+	4'80	+	4'69	- '04	+ '15
				-3	+	'35	+	'35	...	...
γb	0	0	3	2	-	'14	-	'15	...	+ '01
				0	-	6'30	-	6'30	...	...
				-2	-	2'19	-	2'19	...	...
				-4	-	'06	-	'07	...	+ '01
γc^2	2	0	1	4	+	'03	+	'03	...	...
				2	+	1'52	+	1'52	...	...
				0	+	61'91	+	61'90	- '01	+ '02
				-2	-	15'57	-	15'56	...	- '01
				-4	-	'64	-	'63	...	- '01
				-6	-	'08	-	'07	...	- '01
	-2	0	1	6	+	'06	+	'06	...	...
				4	+	2'41	+	2'42	...	- '01
				2	-	1'62	-	1'62	...	...
				0	-	31'76	-	31'77	...	+ '01
				-2	-	2'15	-	2'15	...	...
				-4	-	'05	-	'05	...	...
$\gamma c e'$	1	1	1	4	-	'01	-	'01	...	...
				2	-	'24	-	'24	...	...
				0	-	5'33	-	5'34	+ '01	...
				-2	-	7'46	-	7'48	+ '01	+ '01
				-4	-	'60	-	'52	...	- '08
				-6	-	'02	-	'00	...	- '02

Latitude. Coefficients of Sines.

O.	I.	P.	F.	D.	B.	H.	R.	B-(H+R).	
$\gamma ee'$	-1	-1	1	6	+	'01	+	'01	...
				4	+	'34	+	'35	...
				2	+	8'90	+	8'91	...
				0	+	5'10	+	5'13	...
				-2	+	'83	+	'83	...
				-4	+	'02	+	'01	...
	1	-1	1	4	+	'03	+	'03	...
				2	+	1'14	+	1'14	...
				0	+	6'76	+	6'76	...
				-2	+	'80	+	'80	...
				-4	+	'17	+	'15	...
	-1	1	1	4	-	'05	-	'06	...
				2	-	1'32	-	1'32	...
				0	-	5'66	-	5'67	...
				-2	-	1'77	-	1'78	...
				-4	-	'06	-	'05	...
$\gamma e'2$	0	2	1	2	-	'02	-	'02	...
				0	-	'06	-	'06	...
				-2	-	1'10	-	1'10	...
				-4	-	'03	-	'02	...
	0	-2	1	4	+	'01	+	'01	...
				2	+	'39	+	'39	...
				0	+	'02	+	'02	...
				-2	+	'14	+	'13	...
γea	1	0	1	3	-	'01	-	'01	...
				1	-	'67	-	'65	...
				-1	+	'43	+	'42	...
				-3	+	'31	+	'30	...
				-5	+	'01	+	'00	...
	-1	0	1	3	-	'21	-	'21	...
				1	+	'14	+	'13	...
				-1	+	'59	+	'58	...
				-3	+	'04	+	'04	...
$\gamma e'a$	0	1	1	3	+	'01	+	'01	...
				1	+	'80	+	'79	...
				-1	+	'01	+	'02	...
				-3	+	'03	-	'01	...
	0	-1	1	1	-	'02	-	'01	...
				-1	-	'81	-	'79	...
				-3	-	'04	-	'02	...

Latitude. Coefficients of Sines.

C.	L.	P.	F.	D.	B.	H.	R.	B-(H+R).		
γ^{3e}	1	0	3	2	-	'03	-	'03	...	...
				0	-	1'02	-	1'02	...	...
				-2	-	'33	-	'33	...	...
				-4	+	'01	+	'02	...	- '01
	-1	0	3	4	-	'01	-	'01	...	...
				2	-	'24	-	'25	...	+ '01
				0	-	2'81	-	2'81	...	...
				-2	+	'29	+	'29	...	...
				-4	+	'01		'00	...	+ '01
$\gamma^{3e'}$	0	1	3	0	+	'01	+	'01	...	...
				-2	-	'09	-	'09	...	...
				-4	-	'01		'00	...	- '01
	0-1		3	2	-	'01	-	'01	...	...
				0		'00		'00	...	...
				-2	+	'06	+	'07	...	- '01
γ^{e3}	3	0	1	2	+	'14	+	'14	...	...
				0	+	3'98	+	3'98	...	...
				-2	-	1'52	-	1'52	...	...
				-4	+	'01		'00	...	+ '01
				-6	-	'01		'00	...	- '01
	-3	0	1	6	+	'03	+	'03	...	...
				4	+	'02	+	'02	...	...
				2	+	'26	+	'27	...	'01
				0	-	1'59	-	1'59	...	...
				-2	-	'15	-	'15	...	...
$\gamma^{e2e'}$	2	1	1	2	-	'03	-	'03	...	...
				0	-	'64	-	'64	...	...
				-2	-	'66	-	'66	...	...
				-4	-	'05	-	'03	...	- '02
				-6	-	'01	...	...	...	- '01
	-2-1		1	6	+	'01	+	'01	...	...
				4	+	'22	+	'22	...	...
				2	-	'06	-	'06	...	...
				0	+	31	+	'31	...	...
				-2	+	'06	+	'06	...	...
	2-1		1	2	+	'11	+	'13	...	- '02
				0	+	'81	+	'80	...	+ '01
				-2	-	'08	-	'09	...	+ '01
	-2	1	1	4	-	'03	-	'03	...	...
				2	+	'06	+	'06	...	...
				0	-	'30	-	'31	...	+ '01
				-2	-	'13	-	'12	...	- '01
				-4	-	'01	...	...	...	- '01

Latitude. Coefficients of Sines.

O.	L.	U.	F.	D.	B.	H.	B.	B-(H+B).			
$\gamma_{ee}^{e'2}$	1	2	1	0	-	'06	-	'06	...	...	
			-2	-	'27	-	'28	...	+	'01	
			-4	-	'03	-	'03	...	...	...	
	-1	-2	1	4	+	'02	+	'02	...	...	
			2	+	'32	+	'32	...	...	...	
			0	+	'06	+	'06	...	...	...	
			-2	+	'01	+	'01	...	...	...	
	1	-2	1	2	+	'05	+	'06	...	-	'01
			0	+	'12	+	'12	...	...	...	
			-2	+	'11	+	'10	...	+	'01	
	-1	2	1	2	-	'12	-	'12	...	...	
			0	-	'10	-	'11	...	+	'01	
			-2	-	'07	-	'06	...	-	'01	
$\gamma_{e'}^{e'3}$	0	3	1	-2	-	'04	-	'01	...	-	'03
	0	-3	1	2	+	'01	+	'02	...	-	'01
γ_{3a}	0	0	3	1	+	'01	+	'01	...	...	...
			-1	+	'03	+	'03	...	...	...	...
			-3	+	'01	+	'02	...	-	'01	...
γ_{ee2a}	2	0	1	1	-	'07	-	'06	...	-	'01
			-1	+	'11	+	'11	...	...	...	...
			-3	+	'04	+	'05	...	-	'01	...
			-5	+	'01	+	'00	...	+	'01	...
	-2	0	1	5	-	'01	-	'01	...	...	...
			3	-	'05	-	'05	...	...	...	...
			1	-	'08	-	'07	...	-	'01	...
			-1	+	'04	+	'03	...	+	'01	...
$\gamma_{ee'a}$	1	1	1	1	+	'10	+	'10	...	...	...
			-1	-	'01	-	'02	...	+	'01	...
			-3	+	'02	+	'01	...	+	'01	...
	-1	-1	1	3	-	'01	-	'01	...	...	...
			1		'00	-	'01	...	+	'01	...
			-1	-	'03	-	'01	...	-	'02	...
	1	-1	1	1	-	'01	-	'01	...	...	...
			-1	-	'01	-	'02	...	+	'01	...
			-3	-	'01	-	'00	...	-	'01	...
	-1	1	1	3	+	'02	+	'02	...	...	...
			1	-	'06	-	'05	...	-	'01	...
			-1	+	'02	+	'01	...	+	'01	...
γ^s	0	0	5	0	+	'01	+	'01	...	...	...

Latitude. Coefficients of Sines.

C.	L.	V.	F.	D.	B.	H.	R.	B-(H+R). ¹		
γ_{3e^2}	2	0	3	0	-	'12	-	'12	...	...
				-2	-	'02	-	'02	...	...
				-4	+	'01	+	'01	...	...
	2	0	3	4	-	'01	-	'01	...	...
				2	-	'07	-	'07	...	...
				0	+	'13	+	'11	...	+ '02
				-2	+	'01	+	'01	...	...
γ_{3e^3}	1	1	3	0	+	'01	+	'01	...	...
				-2	-	'01	-	'01	...	...
	-1	-1	3	2	-	'01	-	'01	...	...
				0	+	'01		'00	...	+ '01
				-2	-	'01	-	'01	...	...
	1	-1	3	0	-	'01	-	'01	...	...
	-1	1	3	0	-	'01	-	'01	...	...
				-2	+	'02	+	'03	...	- '01
γ_{e^4}	4	0	1	2	+	'01	+	'01	...	...
				0	+	'27	+	'26	...	+ '01
				-2	-	'14	-	'13	...	- '01
				-4	+	'01	+	'01	...	...
	-4	0	1	2	+	'03	+	'03	...	...
				0	-	'09	-	'09	...	...
				-2	-	'01	-	'01	...	...
$\gamma_{e^3e^2}$	3	1	1	0	-	'06	-	'06	...	...
				-2	-	'06	-	'06	...	...
				-4		'00	+	'01	...	- '01
	-3	-1	1	2	+	'01	+	'01	...	...
				0	+	'02	+	'03	...	- '01
				-2	+	'01	...	...	...	+ '01
	3	-1	1	2	+	'01	+	'01	...	...
				0	+	'08	+	'08	...	...
				-2	-	'02	-	'03	...	+ '01
	-3	1	1	4		'00	+	'01	...	- '01
				0	-	'02	-	'02	...	...
				-2	-	'01	...	...	...	- '01
$\gamma_{e^2e^2}$	2	2	1	0	-	'01	-	'01	...	...
				-2	-	'02	-	'03	...	+ '01
	-2	-2	1	4	+	'01	+	'01	...	...
	2	-2	1	2	+	'01	+	'01	...	...
				0	+	'02	+	'02	...	...
				-2	+	'01	+	'02	...	- '01
	-2	2	1	2	-	'01	-	'01	...	...
				0	-	'01		'00	...	- '01
				-2	-	'01	...	...	...	- '01

Latitude. Coefficients of Sines.

C.	I.	I'.	F.	D.	B.	H.	R.	B-(H+R).		
$\gamma e e^3$	1	3	1	-2	-	'01	-	'01	...	...
	-1	-3	1	2	+	'01	+	'01	...	...
$\gamma e 3 a$	3	0	1	1	-	'01	-	'01	...	...
			-1		+	'01	+	'02	...	-'01
	-3	0	1	1	-	'01	...	...	...	-'01
$\gamma e 2 e' a$	2	1	1	1	+	'01	+	'01	...	...
	2	-1	1	-1	-	'02	-	'02	...	...
	-2	1	1	1	+	'02	+	'02	...	...
$\gamma 3 e 3$	3	0	3	0	-	'01	-	'01	...	...
	-3	0	3	0		'00	+	'01	...	-'01
$\gamma e 5$	5	0	1	0	+	'02	+	'02	...	...
			-2		-	'01	-	'01	...	...
	-5	0	1	0	-	'01	-	'01	...	...
$\gamma e 4 e'$	4	-1	1	0	Not cal.	+	'01	...	...	...
	4	1	1	-2	"	-	'01	...	...	...

Sine Parallax. Coefficients of Cosines.

C.	I.	I'.	F.	D.	B.	H.	R.	B-(H+R).		
I	0	0	0	6	+	'003	+	'002	...	+'001
				4	+	'261	+	'261	...	...
				2	+	28'233	+	28'225	+'005	+'003
				0	+	3422'700	+	3422'09	+'61	...
e	1	0	0	6	+	'001	...	...	...	+'001
				4	+	'043	+	'043	...	...
				2	+	3'086	+	3'084	+'001	+'001
				0	+	186'540	+	186'483	+'029	+'028
				-2	+	34'312	+	34'309	+'005	-'002
				-4	+	'601	+	'599	...	+'002
				-6	+	'009	+	'007	...	+'002
e'	0	1	0	4	-	'005	-	'004	...	-'001
				2	-	'300	-	'301	...	+'001
				0	-	'400	-	'393	...	-'007
				-2	+	1'920	+	1'920	-'002	+'002
				-4	+	'034	+	'035	...	-'001
				-6	+	'001	...	...	...	+'001
a	0	0	0	3	+	'002	+	'003	...	-'001
				1	-	'975	-	'953	+'007	-'029
e ²	2	0	0	4	+	'005	+	'004	...	+'001
				2	+	'283	+	'283	...	...
				0	+	10'166	+	10'161	+'002	+'003
				-2	-	'304	-	'302	...	-'002
				-4	+	'372	+	'372	...	...
				-6	+	'011	+	'010	...	+'001

Sine Parallax. Coefficients of Cosines.

O.	I.	V.	F.	D.	B.	H.	R.	B-(H+R).	
<i>ee'</i>	I	I	O	4	—	'001	...	—'001	
				2	—	'049	...	...	
				0	—	'950	+	'010	
				—2	+	'446	—	'001	
				—4	+	'067	...	—'002	
				—6	+	'002	...	+	'001
	I—I	O	4	+	'006	+	'007	...	—'001
				2	+	'231	...	+	'002
				0	+	'154	—'001	+	'011
				—2	—	'226	...	+	'001
				—4	—	'010	...	—'001	
				—6	—	'001	...	—'001	
<i>e'2</i>	O	2	O	2	—	'003	...	—'001	
				0	—	'009	...	—'001	
				—2	+	'092	...	...	
				—4	+	'003	...	+	'001
<i>γ2</i>	O	O	2	2	—	'001	...	...	
				0	—	'012	...	...	
				—2	—	'105	...	...	
				—4	+	'003	...	+	'001
<i>ea</i>	I	O	O	I	—	'109	+	—'004	
				—I	+	'012	...	+	'001
				—3	—	'039	...	—'002	
<i>e'a</i>	O	I	O	3	+	'003	...	+	'001
				I	+	'149	—'001	+	'004
				—I	—	'004	...	...	
				—3	+	'001	...	...	
<i>e3</i>	3	O	O	4	+	'001	...	+	'001
				2	+	'024	...	+	'001
				0	+	'622	...	+	'002
				—2	—	'119	...	+	'002
				—4	+	'007	...	—'001	
				—6	+	'005	...	+	'001
<i>e2e'</i>	2	I	O	2	—	'005	...	—'001	
				0	—	'104	...	+	'018
				—2	—	'019	...	...	
				—4	+	'032	...	...	
				—6	+	'002	...	+	'001
	2—I	O	4	+	'001	...	...	+	'001
				2	+	'021	...	—'001	
				0	+	'127	...	—'022	
				—2	—	'002	...	...	
				—4	—	'004	...	...	

Sine Parallax. Coefficients of Cosines.

O.	I.	I'.	F.	D.	B.	H.	R.	B-(H+R).		
ee'^2	1	2	0	0	-	'011	-	'010	...	- '001
			-2	+	049	+	'049	...	...	...
			-4	+	'004	+	'004	...	...	...
	1-2	0	4	+	'001	...	...	...	...	+ '001
			2	+	'011	+	'012	...	...	- '001
			0	+	'020	+	'012	...	...	+ '008
			-2	-	'021	-	'021	...	...	...
e'^3	0	3	0-2	+	004	+	'004	...	...	...
$e\gamma^2$	1	0	2	0	-	'001	'000	...	...	- '001
			-2	-	'083	-	'083	...	...	...
			-4	+	'001	+	'001	...	...	...
	1	0-2	4	-	'001	...	...	...	...	- '001
			2	-	'048	-	'048	...	...	...
			0	-	'714	-	'709	...	...	- '005
			-2	-	'011	-	'011	...	...	...
$e'\gamma^2$	0	1	2	0	+	001	+	'001	...	...
			-2	-	'007	-	'007	...	...	...
			-4	+	'001	...	...	...	...	+ '001
	0	1-2	2	+	'001	+	'001	...	...	...
			0	+	'002	+	'001	...	...	+ '001
e^2a	2	0	0	1	-	'010	-	'010	...	...
			-1	+	'016	+	'015	...	...	+ '001
			-3	-	'009	-	'009	...	...	...
			-5	-	'001	...	...	...	...	- '001
$ee'a$	1	1	0	1	+	'016	+	'016	...	...
			-1		'000	...	...	...	...	...
			-3	-	'003	-	'002	...	...	- '001
	1-1	0	1	-	'001	-	'001	...	...	...
			-1		'000	-	'001	...	...	+ '001
			-3	+	'004	+	'003	...	...	+ '001
γ^2a	0	0	2-1	+	'007	+	'007	...	...	...
			-3	-	'002	-	'002	...	...	...
e^4	4	0	0	2	+	'002	+	'001	...	+ '001
			0	+	'040	+	'040	...	...	...
			-2	-	'013	-	'014	...	...	+ '001
e^3e'	3	1	0	2	-	'001	...	...	...	- '001
			0	-	'010	-	'010	...	...	...
			-2	-	'005	-	'004	...	...	- '001
			-4	+	'001	...	...	...	...	+ '001
			-6	+	'001	...	...	...	...	+ '001
	3-1	0	2	+	'002	+	'001	...	...	+ '001
			0	+	'012	+	'012	...	...	...
			-2	-	'002	-	'001	...	...	- '001

Sine Parallax. Coefficients of Cosines.

U.	I.	P.	F.	D.	B.	H.	R.	B-(H+R).
$e^2e'^2$	2	2	0	0	—	'001	...	—'001
				-2	—	'001	—	'001
				-4	+	'002	+	'002
	2-2	0	2	+	'001	+	'001	...
			0	+	'002	+	'001	...
			-2		'000	+	'001	...
			-4	—	'001		'000	...
ee'^3	1	3	0-2	+	'001	+	'001	...
$e^2\gamma^2$	2	0	2-2	—	'009	—	'009	...
	2	0-2	2	—	'005	—	'004	...
			0		'000		'000	...
			-2	—	'014	—	'014	...
$ee'\gamma^2$	1	1	2-2	—	'003	—	'002	...
	1	1-2	2	+	'001	+	'000	...
			0	+	'002	+	'002	...
			-2	—	'001		...	...
	1-1	2-2			'000	+	'001	...
	1-1-2	2		+	'003	—	'003	...
			0	—	'003	—	'003	...
e^3a	3	0	0	1	—	'001	—	'001
			-1	+	'002	+	'001	...
$e^2e'a$	2	1	0	1	+	'002	+	'001
			-1		'000	...	...	...
			-3	—	'001	...	...	...
	2-1	0	1	—	'001	...	...	...
			-1	—	'003	—	'003	...
			-3	—	'001	...	...	...
ek^2a	1	0	2-1	+	'001	...	...	...
	1	0-2-1		+	'001	...	...	...
e^5	5	0	0	0	+	'003	+	'003
			-2	—	'001	—	'002	...
e^4e'	4	1	0	0	Not cal.	—	'001	...
			-2		„	—	'001	...
	4-1	0	0		„	+	'001	...
e^3k^2	3	0	2-2	—	'001	—	'001	...
	3	0-2	2	—	'001	...	...	...
			-4	—	'001	...	...	...

Postscript added 1905 January 25.—The coefficients resulting from the theory are given above to hundredths of a second in longitude and latitude, and to thousandths of a second in parallax. In each case they have actually been calculated to one more place of decimals. Further, the coefficients, referred to rectangular coordinates, have been found with even greater accuracy in the great majority of cases. The final results with the full number of places of decimals used are probably correct within two units of the last figure given, as far as the terms in any one characteristic are concerned ; they will be found in the fourth part of the "Theory of the Motion of the Moon," shortly to appear in the *Memoirs of the Society*.

The coefficients are all definitive with the exception of two or three, which require small corrections not exceeding $0''.02$, due to certain terms of the disturbing function which have been neglected. See *Mem. R.A.S.* vol. liii. p. 50.

Haverford College :

1904 November 15.

(Communicated by the Astronomer-Royal.)

Day.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
1904.						
Jan. 26 (a) (b)	Disapp. W. B. II. 1033	Astrographic Equatorial	225	Dark	7 18 29.76	H.
26 (b)	"	Thompson Equatorial	110	"	7 18 30.25	C. D.
26 (b)	"	Great Equatorial	670	"	7 18 30.40	W. B.
26 (b)	"	Merz Refractor	250	"	7 18 29.85	P. M.
Mar. 21 (c)	" W. B. III. 474	Astrographic Equatorial	225	"	9 33 53.47	W. S.
22 (d) (e)	" 61 Tauri	"	225	"	9 59 34.22	H.
22 (d)	"	Sheepshanks Equatorial	100	"	9 59 34.36	W.
22 (d) (e)	" 75 Tauri	Astrographic Equatorial	225	"	10 2 51.87	H.
22 (d)	"	Sheepshanks Equatorial	100	"	10 2 52.21	W.
22 (d) (e)	" W. B. (2) IV. 450	Astrographic Equatorial	225	"	10 3 38.24	H.
22 (d)	"	Sheepshanks Equatorial	100	"	10 3 37.28	W.
27	" W. B. IX. 127	Astrographic Equatorial	225	"	8 17 5.54	W. S.
27 (e)	" Piazzi IX. 35	"	225	"	9 47 22.37	W. S.
Apr. 19	" B. D. + 16°, 701	"	225	"	8 33 40.79	W. S.
19	" Lalande 9625	"	225	"	8 34 52.59	W. S.
21	" B. D. + 17°, 1488	"	225	"	9 36 51.28	W. S.
21	" B. D. + 16°, 1373	"	225	"	9 41 4.59	W. S.
21	" W. B. (2) IV. 1723-4	"	225	"	9 57 30.89	W. S.

Day.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
1904. Apr. 24	Disapp. 10 Sextantis	Great Equatorial	670	Dark	9 24 1'71	H. F.
24	"	Great Equatorial (Corbett)	120	"	9 24 2'01	V.
July 9	Reapp. θ^2 Tauri	"	120	"	14 56 12'45	D.
9	" θ^1 Tauri	Astrographic Equatorial	225	"	14 57 50'72	H.
9	"	Sheepshanks Equatorial	100	"	14 57 50'30	A. C.
9	Disapp. Bradley 619	Great Equatorial (Corbett)	120	Bright	14 59 (59'33)	D.
9 (f)	"	Astrographic Equatorial	225	"	15 0 1'17	H.
9 (f)	"	Great Equatorial	670	"	15 0 1'13	W. B.
9	" Aldebaran	" (Corbett)	120	"	17 31 2'43	D.
9 (a)	"	Astrographic Equatorial	225	"	17 31 2'27	H.
9 (a) (i)	"	Sheepshanks Equatorial	100	"	17 31 3'07	A. C.
9	"	Great Equatorial	670	"	17 31 2'53	W. B.
9	Reapp.	" (Corbett)	120	Dark	18 24 27'06	D.
9	"	"	670	"	18 24 26'76	W. B.
Aug. 27 (g)	" 20 Piscium	Sheepshanks Equatorial	100	"	11 15 (51'32)	V.
27 (g)	"	Great Equatorial	670	"	11 15 (48'63)	J.
30	" ξ^1 Ceti	Sheepshanks Equatorial	100	"	16 12 37'94	W.
Sept. 19 (e) (h)	Disapp. B. D. - 16°, 5609	Merz Refractor	250	"	11 29 34'77	C. D.
29 (c)	" γ Tauri	Astrographic Equatorial	225	Bright	9 16 47'42	W.
29	Reapp.	Sheepshanks Equatorial	100	Dark	10 2 19'60	H. F.

Day.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
Sept. 29 (a)	1904.					
29	Reapp. γ Tauri	Astrographic Equatorial	225	Dark	10 2 20.21	W.
29 (e)	Disapp. η Tauri	Great Equatorial	670	Bright	12 33 33.86	D.
29 (g) (j)	"	Astrographic Equatorial	225	"	12 33 (30.93)	H.
29	"	Merz Refractor	250	"	12 33 32.61	C. D.
29	"	Thompson Equatorial	110	"	12 33 (36.11)	R. F.
29	Reapp.	Great Equatorial	670	Dark	13 7 41.30	D.
29 (a)	"	Astrographic Equatorial	225	"	13 7 40.66	H.
29	"	Merz Refractor	250	"	13 7 40.97	C. D.
29	"	Sheepshanks Equatorial	100	"	13 7 40.68	W.
29	"	Thompson Equatorial	110	"	13 7 41.17	R. F.
29 (i)	Disapp. θ Tauri	Great Equatorial	670	Bright	13 40 2.64	D.
29 (e) (c)	"	Astrographic Equatorial	225	"	13 39 (58.99)	H.
29 (g)	"	Merz Refractor	250	"	13 40 4.12	C. D.
29 (g)	"	Sheepshanks Equatorial	100	"	13 39 (59.60)	W
29	"	Thompson Equatorial	110	"	13 39 (56.82)	R. F.
29	θ Tauri	Great Equatorial	670	"	13 43 48.70	D.
29 (e)	"	Astrographic Equatorial	225	"	13 43 47.56	H.
29	"	Merz Refractor	250	"	13 43 48.49	C. D.
29	"	Sheepshanks Equatorial	100	"	13 43 47.48	W.
29	"	Thompson Equatorial	110	"	13 43 45.99	R. F.

Day.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
1904. Sept. 29 (g)	Disapp. W. B. (2) IV. 450	Great Equatorial	670	Bright	14 8 20.44	D.
29 (c) (g)	"	Astographic Equatorial	225	"	14 8 (10.86)	H.
29	Reapp. θ_2 Tauri	Great Equatorial	670	Dark	14 53 51.40	D.
29 (a)	"	Astographic Equatorial	225	"	14 53 51.24	H.
29	"	Merz Refractor	250	"	14 53 51.12	C. D.
29	"	Sheepshanks Equatorial	100	"	14 53 51.74	W.
29	"	Thompson Equatorial	110	"	14 53 50.72	R. F.
29	" θ_1 Tauri	Great Equatorial	670	"	14 57 44.06	D.
29 (a)	"	Astographic Equatorial	225	"	14 57 44.71	H.
29	"	Merz Refractor	250	"	14 57 43.98	C. D.
29	"	Sheepshanks Equatorial	100	"	14 57 44.42	W.
29	"	Thompson Equatorial	110	"	14 57 44.18	R. F.
29	Disapp. Bradley 619	Great Equatorial	670	Bright	15 1 8.99	D.
29 (c) (f)	"	Astographic Equatorial	225	"	15 1 (5.75)	H.
29	"	Merz Refractor	250	"	15 1 8.71	C. D.
29 (c) (g)	"	Sheepshanks Equatorial	100	"	15 1 8.35	W.
29 (l)	Reapp. W. B. (2) IV. 450	Great Equatorial	670	Dark	15 12 (29.34)	D.
29 (c) (l)	"	Merz Refractor	250	"	15 12 (17.37)	C. D.
29 (l)	"	Sheepshanks Equatorial	100	"	15 12 (23.13)	W.
29 (l)	"	Thompson Equatorial	110	"	15 12 (17.37)	R. F.

Day.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
1904. Nov. 20	Disapp. 64 Ceti	Sheepshanks Equatorial	100	Dark	9 12 33.97	J. S.
20	"	Astrographic Equatorial	225	"	9 12 33.79	W. S.
20	"	Sheepshanks Equatorial	100	"	10 23 26.54	J. S.
20	"	Astrographic Equatorial	225	"	10 23 26.35	W. S.
Dec. 20	"	Great Equatorial (Corbett)	120	"	6 0 50.10	D.
20 (a)	"	Astrographic Equatorial	225	"	6 0 50.12	H.
20	"	Merz Refractor	250	"	6 0 50.03	C. D.
20	"	Great Equatorial	670	"	6 0 50.30	W. B.
20	"	Sheepshanks Equatorial	100	"	6 0 50.00	H. F.
20	Reapp. "	Great Equatorial	670	Bright	7 4 37.61	W. B.
20	Disapp. 70 Tauri	Thompson Equatorial	110	Dark	9 19 26.16	C. D.
20	"	Great Equatorial	670	"	9 19 26.58	W. B.
20	"	" (Corbett)	120	"	11 16 0.52	D.
20	"	Astrographic Equatorial	225	"	11 16 1.10	H.
20	"	Merz Refractor	250	"	11 16 0.77	C. D.
20	"	Great Equatorial	670	"	11 16 0.52	W. B.
20	"	Thompson Equatorial	110	"	11 16 0.37	R. F.
20	"	Old Altazimuth	100	"	11 16 1.29	V.
20	"	Great Equatorial (Corbett)	120	"	11 23 13.34	D.
20	"	Astrographic Equatorial	225	"	11 23 13.22	H.

Day.	Phenomenon.	Telescope.	Power.	Moon's Limb.	Mean Solar Time of Observation. h m s	Observer.
1904 Dec. 20	Disapp. W. B. (2) IV. 450	Merz Refractor	250	Dark	11 23 13.78	C. D.
20	"	Great Equatorial	670	"	11 23 13.44	W. B.
20 (e) (f) (k)	" ϵ Tauri	Astrographic Equatorial	225	"	11 24 38.89	H.
20 (k)	"	Merz Refractor	250	"	11 24 39.65	C. D.
20 (k)	"	Great Equatorial	670	"	11 24 39.41	W. B.
20 (k)	"	Sheepshanks Equatorial	100	"	11 24 38.52	H. F.
20 (k)	"	Old Altazimuth	100	"	11 24 (31.37)	V.
20	Reapp. "	Great Equatorial	670	Bright	11 57 48.66	D
20	Disapp. Bradley 619	"	670	Dark	12 20 15.47	D.
20 (a)	"	Astrographic Equatorial	225	"	12 20 15.97	H.
20	"	Merz Refractor	250	"	12 20 15.90	C. D.
20	"	Great Equatorial (Corbett)	120	"	12 20 15.97	W. B.
20	"	Sheepshanks Equatorial	100	"	12 20 15.68	R. C.
20	"	Great Equatorial (Corbett)	120	"	15 18 26.50	D.
20	Aldebaran	Astrographic Equatorial	225	"	15 18 26.43	H.
20 (a)	"	Merz Refractor	250	"	15 18 26.38	C. D.
20	"	Great Equatorial	670	"	15 18 26.60	W. B.
20	"	Sheepshanks Equatorial	100	"	15 18 25.94	E.
20	"	Thompson Equatorial	110	"	15 18 26.18	R. F.

Notes.

- (a) Instantaneous. (b) Unsteady, cloudy. (c) Very faint. (d) Diffused. (e) Not quite instantaneous.
 (f) The star appeared to glide up to and under the limb. (g) Observation doubtful. (h) Definition not good.
 (i) The star was projected on the limb of the Moon. (j) The limb was boiling.
 (k) This phenomenon was very near the bright limb of the Moon.
 (l) The times of observation differ so widely that no reliable mean can be adopted.

The apertures of the telescopes used are as follows :—

	inches.		inches.
Great Equatorial ...	... 28	Sheepshanks Equatorial ...	... $6\frac{3}{4}$
Thompson Equatorial ...	... 26	Great Equatorial (Corbett Telescope) ...	... $6\frac{1}{2}$
Merz Refractor ...	... $12\frac{3}{4}$	Old Altazimuth ...	... 4
Astrographic Equatorial (Guiding Telescope) ...	... 10		

The initials D., H., A. C., C. D., W. B., H. F., W., J. S., P. M., W. S., E., R. C., R. F., V., J., are those of Mr. Dyson, Mr. Hollis, Mr. Crommelin, Mr. Davidson, Mr. Bowyer, Mr. Furner, Mr. Witchell, Mr. Storey, Mr. Melotte, Mr. Stevens, Mr. J. Evans, Mr. Cullen, Mr. Fowler, Mr. Vagg, and Mr. James respectively.

It may be pointed out that the *Nautical Almanac* predictions for the star θ^2 *Tauri* are erroneous throughout the year. The R.A. and Dec. of this star, as given in the *Nautical Almanac* for 1904, are too great by $10^s.18, 25''.3$ respectively.

Royal Observatory, Greenwich:
 1905 January 13.

Ephemeris for Physical Observations

Greenwich Mean Noon. 1905.	P.	L-O.	B.	Equat. Diam.	Excess over Polar.	Defect of Illum.	Q.	d.
June 8	344°35	275°81	+ 2°97	33°94	2°19	0°06	253°06	5°06
13	344°75	276°93	2°98	34°14	2°20	°08	253°67	5°72
18	345°14	278°02	2°99	34°36	2°21	°10	254°25	6°36
23	345°54	279°09	3°00	34°61	2°23	°13	254°77	6°98
28	345°93	280°14	3°01	34°90	2°25	°15	255°27	7°58
July 3	346°32	281°17	3°02	35°21	2°27	°18	255°75	8°15
8	346°69	282°17	3°03	35°54	2°29	°20	256°21	8°69
13	347°05	283°12	3°04	35°91	2°31	°23	256°64	9°19
18	347°41	284°03	3°04	36°31	2°34	°25	257°05	9°66
23	347°75	284°90	3°05	36°74	2°37	°28	257°45	10°08
28	348°09	285°74	3°06	37°19	2°40	°31	257°83	10°46
Aug. 2	348°41	286°53	3°06	37°67	2°43	°33	258°19	10°79
7	348°71	287°26	3°07	38°18	2°46	°35	258°53	11°08
12	348°99	287°94	3°08	38°72	2°49	°37	258°86	11°31
17	349°24	288°56	3°09	39°29	2°53	°39	259°16	11°48
22	349°47	289°12	3°10	39°88	2°57	°41	259°44	11°59
27	349°67	289°61	3°11	40°49	2°61	°41	259°70	11°63
Sept. 1	349°84	290°03	3°12	41°13	2°65	°42	259°93	11°60
6	349°99	290°37	3°13	41°78	2°69	°42	260°14	11°50
11	350°11	290°64	3°13	42°45	2°74	°41	260°33	11°32
16	350°20	290°85	3°14	43°12	2°78	°40	260°49	11°08
21	350°25	290°97	3°15	43°80	2°82	°38	260°63	10°74
26	350°25	290°99	3°16	44°48	2°86	°36	260°74	10°32
Oct. 1	350°23	290°93	3°16	45°15	2°91	°33	260°84	9°84
6	350°18	290°80	3°17	45°80	2°95	°30	260°93	9°27
11	350°08	290°58	3°17	46°42	2°99	°26	261°00	8°59
16	349°95	290°27	3°18	47°02	3°03	°22	261°07	7°83
21	349°79	289°89	3°18	47°57	3°07	°18	261°15	7°00
26	349°61	289°44	3°18	48°05	3°10	°14	261°27	6°11
31	349°40	288°93	3°18	48°47	3°12	°10	261°50	5°16
Nov. 5	349°17	288°36	3°18	48°84	3°14	°07	261°88	4°15
10	348°92	287°75	3°18	49°11	3°16	°04	262°63	3°09
15	348°65	287°10	3°17	49°30	3°17	°01	264°47	2°00
20	348°37	286°42	3°16	49°39	3°17	°00	271°40	0°89
25	348°09	285°74	3°15	49°39	3°18	°00	40°57	0°32
30	347°82	285°06	+ 3°14	49°29	3°17	0°01	70°10	1°39

of Jupiter, 1905-6, By A. C. D. Crommelin.

Greenwich Mean Noon. 1905.	Light Time. m	Longitude of Central Meridian.		Corr. for Phase.	A-O.	B.
		I. 877° 90.	II. 870° 27.			
June 8	48° 945	296° 85	293° 56	+ 0° 11	270° 75	+ 3° 07
13	48° 663	5° 40	323° 96	14	271° 21	3° 07
18	48° 345	74° 00	354° 41	18	271° 66	3° 07
23	47° 993	142° 64	24° 90	21	272° 11	3° 07
28	47° 606	211° 33	55° 44	25	272° 56	3° 07
July 3	47° 186	280° 05	86° 01	29	273° 02	3° 06
8	46° 737	348° 83	116° 63	33	273° 48	3° 06
13	46° 259	57° 67	147° 32	37	273° 93	3° 06
18	45° 753	126° 57	178° 07	41	274° 37	3° 06
23	45° 223	195° 52	208° 86	44	274° 82	3° 06
28	44° 669	264° 52	239° 71	48	275° 28	3° 06
Aug. 2	44° 096	333° 58	270° 62	51	275° 74	3° 05
7	43° 504	42° 71	301° 60	54	276° 18	3° 05
12	42° 900	111° 90	332° 64	56	276° 63	3° 05
17	42° 281	181° 16	3° 74	57	277° 08	3° 04
22	41° 656	250° 48	34° 91	58	277° 53	3° 04
27	41° 024	319° 88	66° 15	59	277° 98	3° 04
Sept. 1	40° 393	29° 34	97° 46	58	278° 43	3° 03
6	39° 762	98° 88	128° 85	57	278° 87	3° 03
11	39° 138	168° 49	160° 30	56	279° 32	3° 03
16	38° 524	238° 16	191° 82	54	279° 77	3° 02
21	37° 927	307° 92	223° 33	50	280° 23	3° 02
26	37° 349	17° 74	254° 89	46	280° 67	3° 02
Oct. 1	36° 797	87° 63	286° 73	42	281° 09	3° 01
6	36° 271	157° 58	318° 63	37	281° 53	3° 01
11	35° 783	227° 61	350° 50	32	281° 99	3° 00
16	35° 332	297° 68	22° 42	27	282° 44	3° 00
21	34° 926	7° 81	54° 40	21	282° 89	2° 99
26	34° 568	77° 98	86° 42	16	283° 33	2° 99
31	34° 263	148° 18	118° 46	12	283° 77	2° 98
Nov. 5	34° 014	218° 40	150° 53	07	284° 22	2° 97
10	33° 826	288° 63	182° 61	04	284° 67	2° 97
15	33° 698	358° 85	214° 68	+ 02	285° 10	2° 96
20	33° 634	69° 06	246° 74	00	285° 54	2° 95
25	33° 634	139° 25	278° 78	00	286° 00	2° 95
30	33° 701	209° 39	310° 77	+ 00	286° 44	+ 2° 94

Greenwich Mean Noon. 1905.	P.	L-O.	B.	Equat. Diam.	Excess over Polar.	Defect of Illum.	Q.	d.
Dec. 5	347°56	284°40	+ 3°12	49°10	3°16	0°02	73°50	2°49
10	347°32	283°77	3°10	48°82	3°14	°05	74°71	3°56
15	347°09	283°18	3°08	48°45	3°12	°08	75°28	4°59
20	346°88	282°64	3°06	48°01	3°09	°11	75°59	5°57
25	346°69	282°17	3°03	47°50	3°06	°15	75°73	6°48
30	346°53	281°76	3°01	46°93	3°02	°19	75°79	7°33
1906. Jan. 4	346°41	281°43	2°99	46°32	2°98	°23	75°85	8°12
9	346°31	281°19	2°97	45°66	2°94	°27	75°95	8°80
14	346°25	281°02	2°94	44°99	2°90	°30	76°05	9°40
19	346°22	280°94	2°92	44°29	2°86	°33	76°16	9°93
24	346°22	280°95	2°90	43°58	2°81	°35	76°27	10°36
29	346°25	281°04	2°88	42°87	2°76	°37	76°39	10°71
Feb. 3	346°32	281°22	2°86	42°17	2°71	°38	76°52	10°97
8	346°41	281°48	2°84	41°48	2°67	°39	76°68	11°15
13	346°54	281°81	2°82	40°79	2°63	°39	76°84	11°25
18	346°70	282°23	2°80	40°12	2°58	°39	77°02	11°28
23	346°88	282°71	2°79	39°48	2°54	°38	77°21	11°23
28	347°09	283°27	2°77	38°87	2°50	°37	77°44	11°12
Mar 5	347°33	283°88	2°76	38°28	2°46	°35	77°69	10°95
10	347°60	284°56	2°74	37°72	2°42	°33	77°99	10°71
15	347°89	285°29	2°73	37°19	2°39	°31	78°31	10°41
20	348°20	286°08	2°71	36°69	2°36	°28	78°65	10°06
25	348°54	286°91	2°70	36°22	2°33	°26	79°01	9°67
30	348°89	287°78	2°69	35°78	2°30	°23	79°38	9°23
Apr. 4	349°27	288°70	2°68	35°37	2°28	°21	79°77	8°75
9	349°66	289°65	2°66	34°99	2°25	°18	80°18	8°24
14	350°08	290°64	2°65	34°64	2°23	°16	80°60	7°69
19	350°51	291°65	2°64	34°33	2°21	°13	81°05	7°11
24	350°95	292°70	2°63	34°04	2°19	°11	81°53	6°51
29	351°41	293°77	2°61	33°78	2°17	°09	82°03	5°88
May 4	351°88	294°85	2°60	33°55	2°16	°07	82°84	5°23
July 15	359°33	311°21	2°36	33°28	2°14	°06	267°93	4°89
20	359°83	312°29	2°34	33°47	2°15	°08	268°45	5°54
25	0°33	313°36	2°32	33°68	2°17	°10	268°97	6°18
30	0°82	314°40	2°29	33°92	2°18	°12	269°47	6°79
Aug. 4	1°29	315°42	2°27	34°19	2°20	°15	269°97	7°37
9	1°75	316°40	+ 2°25	34°49	2°22	0°17	270°46	7°93

Jan. 1905.

Observations of Jupiter, 1905-6.

307

Greenwich Mean Noon. 1905.	Light time. m	Longitude of Central Meridian.		Corr. for Phase.	A-O.	B.
		I. 877° 90.	II. 870° 27.			
Dec. 5	33' 8.32	279° 47'	342° 70'	-0° 03'	286° 89'	+2° 94'
10	34' 0.29	349° 48'	14° 56'	0° 06'	287° 33'	2° 93'
15	34' 2.86	59° 41'	46° 35'	0° 09'	287° 77'	2° 92'
20	34' 6.02	129° 25'	78° 04'	0° 14'	288° 21'	2° 92'
25	34' 9.71	199° 00'	109° 64'	0° 18'	288° 65'	2° 91'
30	35' 3.94	268° 65'	141° 14'	0° 23'	289° 10'	2° 90'
Jan. 4	35' 8.64	338° 20'	172° 54'	0° 29'	289° 54'	2° 89'
9	36' 3.77	47° 63'	203° 82'	0° 34'	289° 98'	2° 88'
14	36' 9.27	116° 96'	235° 01'	0° 38'	290° 42'	2° 87'
19	37' 5.08	186° 18'	266° 08'	0° 43'	290° 87'	2° 87'
24	38' 1.18	255° 30'	297° 06'	0° 47'	291° 31'	2° 86'
29	38' 7.50	324° 32'	327° 93'	0° 50'	291° 75'	2° 85'
Feb. 3	39' 3.99	33° 25'	358° 71'	0° 53'	292° 19'	2° 84'
8	40' 0.61	102° 09'	29° 41'	0° 54'	292° 63'	2° 83'
13	40' 7.30	170° 85'	60° 02'	0° 55'	293° 06'	2° 82'
18	41' 4.03	239° 53'	90° 55'	0° 55'	293° 51'	2° 81'
23	42' 0.74	308° 13'	121° 01'	0° 55'	293° 94'	2° 80'
28	42' 7.39	16° 68'	151° 41'	0° 54'	294° 39'	2° 79'
Mar. 5	43' 3.97	85° 16'	181° 74'	0° 52'	294° 83'	2° 78'
10	44' 0.42	153° 59'	212° 02'	0° 50'	295° 27'	2° 77'
15	44' 6.70	221° 97'	242° 26'	0° 47'	295° 70'	2° 76'
20	45' 2.80	290° 31'	272° 45'	0° 44'	296° 14'	2° 75'
25	45' 8.68	358° 62'	302° 61'	0° 41'	296° 58'	2° 74'
30	46' 4.32	66° 90'	332° 75'	0° 37'	297° 01'	2° 73'
Apr. 4	46' 9.69	135° 16'	2° 86'	0° 33'	297° 45'	2° 72'
9	47' 4.77	203° 40'	32° 95'	0° 29'	297° 89'	2° 71'
14	47' 9.53	271° 62'	63° 02'	0° 26'	298° 33'	2° 70'
19	48' 3.97	339° 83'	93° 09'	0° 22'	298° 76'	2° 69'
24	48' 8.07	48° 04'	123° 15'	0° 18'	299° 21'	2° 67'
29	49' 1.81	116° 24'	153° 21'	0° 16'	299° 65'	2° 66'
May 4	49' 5.18	184° 45'	183° 27'	-0° 12'	300° 08'	2° 65'
July 15	49' 9.22	16° 65'	186° 10'	+0° 10'	306° 32'	2° 47'
20	49' 6.40	85° 24'	216° 54'	0° 13'	306° 75'	2° 46'
25	49' 3.22	153° 86'	247° 02'	0° 17'	307° 18'	2° 44'
30	48' 9.68	222° 53'	277° 54'	0° 20'	307° 62'	2° 43'
Aug. 4	48' 5.80	291° 25'	308° 10'	0° 24'	308° 05'	2° 42'
9	48' 1.60	0° 03'	338° 72'	+0° 28'	308° 48'	+2° 40'

Greenwich Mean Noon. 1906.	P.	L-O.	B.	Equat. Diam.	Excess over Polar.	Defect of Illum.	Q.	d.
Aug. 14	2°20	317°36	+2°23	34"82	2"24	0"19	270°93	8°45
19	2°62	318°28	2°21	35°18	2°26	°22	271°39	8°94
24	3°03	319°16	2°19	35°56	2°29	°24	271°82	9°39
29	3°42	320°00	2°17	35°98	2°31	°26	272°22	9°81
Sept. 3	3°79	320°80	2°15	36°42	2°34	°28	272°59	10°17
8	4°14	321°55	2°13	36°89	2°37	°30	272°93	10°49
13	4°46	322°24	2°11	37°38	2°41	°33	273°25	10°76
18	4°75	322°88	2°09	37°90	2°44	°35	273°54	10°98
23	5°02	323°47	2°08	38°45	2°48	°36	273°80	11°13
28	5°25	323°98	2°06	39°02	2°51	°37	274°04	11°22
Oct. 3	5°46	324°43	2°04	39°61	2°55	°38	274°25	11°24
8	5°63	324°81	2°03	40°23	2°59	°38	274°44	11°19
13	5°77	325°12	2°02	40°86	2°63	°38	274°60	11°08
18	5°87	325°35	2°01	41°50	2°67	°37	274°72	10°88
23	5°94	325°50	2°00	42°14	2°71	°36	274°82	10°60
28	5°97	325°56	1°99	42°79	2°75	°34	274°86	10°24
Nov. 2	5°96	325°54	1°98	43°43	2°80	°32	274°87	9°79
7	5°92	325°44	1°98	44°06	2°84	°29	274°84	9°27
12	5°84	325°26	1°97	44°68	2°88	°26	274°79	8°66
17	5°72	325°00	1°97	45°26	2°91	°22	274°71	7°98
22	5°56	324°65	1°97	45°80	2°95	°19	274°60	7°21
27	5°37	324°24	1°97	46°29	2°98	°15	274°46	6°37
Dec. 2	5°15	323°75	1°97	46°74	3°01	°11	274°31	5°46
7	4°91	323°21	1°97	47°11	3°03	°07	274°15	4°50
12	4°64	322°62	1°97	47°41	3°05	°04	273°98	3°49
17	4°35	321°99	1°97	47°63	3°07	°02	273°88	2°43
22	4°04	321°33	1°97	47°77	3°08	°01	274°04	1°35
27	3°73	320°65	1°97	47°81	3°08	°00	278°2	0°25
1907. Jan. 1	3°42	319°97	+1°97	47°76	3°08	0°00	91°10	0°85

Greenwich Mean Noon. 1906.	Light time. m	Longitude of Central Meridian.		Corr. for Phase.	A-O.	B.
		I. 877° 90.	II. 870° 27.			
Aug. 14	47.708	68.85	9.39	+ 0.31	308.91	+ 2.39
19	47.225	137.72	40.11	.35	309.34	2.37
24	46.715	206.65	70.89	.38	309.77	2.36
29	46.178	275.64	101.72	.42	310.19	2.34
Sept. 3	45.619	344.68	132.61	.45	310.63	2.33
8	45.040	53.79	163.57	.48	311.06	2.31
13	44.442	122.96	194.59	.50	311.49	2.30
18	43.829	192.18	225.66	.53	311.91	2.28
23	43.203	261.48	256.80	.54	312.34	2.27
28	42.570	330.85	288.02	.55	312.76	2.25
Oct. 3	41.932	40.30	319.31	.55	313.19	2.24
8	41.294	109.80	350.66	.55	313.62	2.22
13	40.660	179.38	22.09	.54	314.05	2.20
18	40.034	249.03	53.59	.52	314.47	2.19
23	39.420	318.76	85.16	.49	314.90	2.17
28	38.822	28.56	116.81	.46	315.32	2.16
Nov. 2	38.246	98.43	148.53	.42	315.75	2.14
7	37.699	168.36	180.31	.37	316.17	2.12
12	37.183	238.36	212.15	.33	316.60	2.11
17	36.704	308.42	244.05	.28	317.02	2.09
22	36.268	18.53	276.01	.23	317.45	2.07
27	35.881	88.68	308.01	.18	317.87	2.06
Dec. 2	35.543	158.87	340.05	.13	318.29	2.04
7	35.261	229.09	12.12	.09	318.71	2.02
12	35.037	299.31	44.19	.05	319.14	2.01
17	34.876	9.54	76.27	+ .03	319.56	1.99
22	34.779	79.76	108.33	.00	319.98	1.97
27	34.747	149.96	140.38	.00	320.40	1.95
1907. Jan. 1	34.781	220.12	172.39	+ 0.00	320.82	+ 1.94

The following is a list of the Greenwich Mean Times when the adopted zero-meridians of the two systems will pass the middle of the illuminated disc, and the intervals between successive passages, to facilitate the determination of intermediate ones :—

SYSTEM I.

			Interval between Passages 9 ^h + m				Interval between Passages 9 ^h + m				Interval between Passages 9 ^h + m
1905. d	h	m		1905. d	h	m		1905. d	h	m	
June 8	1	43.43	50.63	Aug. 22	22	39.73	50.52	Nov. 6	19	13.73	
10	2	56.55		24	23	52.30		8	20	25.76	
12	4	9.66		27	1	4.86		10	21	37.79	
14	5	22.76		29	2	17.40		12	22	49.82	50.41
16	6	35.84		31	3	29.92		15	0	1.85	
18	7	48.91	50.61	Sept. 2	4	42.42		17	1	13.88	
20	9	1.97		4	5	54.90		19	2	25.92	
22	10	15.02		6	7	7.36		21	3	37.97	
24	11	28.05		8	8	19.81		23	4	50.02	
26	12	41.08		10	9	32.24		25	6	2.08	
28	13	54.09	50.60	12	10	44.65	50.48	27	7	14.15	
30	15	7.09		14	11	57.04		29	8	26.23	
July 2	16	20.07		16	13	9.42		Dec. 1	9	38.31	
4	17	33.04		18	14	21.77		3	10	50.40	50.43
6	18	45.99		20	15	34.12		5	12	2.54	
8	19	58.93		22	16	46.45		7	13	14.71	
10	21	11.86		24	17	58.76		9	14	26.91	
12	22	24.78	50.58	26	19	11.05		11	15	39.13	
14	23	37.67		28	20	23.33		13	16	51.37	
17	0	50.55		30	21	35.59		15	18	3.63	
19	2	3.42		Oct. 2	22	47.83	50.45	17	19	15.91	
21	3	16.27		5	0	0.05		19	20	28.22	
23	4	29.10		7	1	12.26		21	21	40.56	
25	5	41.92		9	2	24.45		23	22	52.93	50.48
27	6	54.73		11	3	36.62		26	0	5.33	
29	8	7.52		13	4	48.78		28	1	17.77	
31	9	20.29		15	6	0.92		30	2	30.23	
Aug. 2	10	33.05	50.55	17	7	13.05		1906. Jan. 1	3	42.72	
4	11	45.79		19	8	25.17		3	4	55.23	
6	12	58.51		21	9	37.27		5	6	7.77	
8	14	11.22		23	10	49.37	50.42	7	7	20.34	
10	15	23.92		25	12	1.44		9	8	32.94	
12	16	36.60		27	13	13.50		11	9	45.57	
14	17	49.26		29	14	25.55		13	10	58.23	50.54
16	19	1.90		31	15	37.60		15	12	10.92	
18	20	14.53		Nov. 2	16	49.65		17	13	23.64	
20	21	27.14		4	18	1.69		19	14	36.39	

SYSTEM I.

			Interval between Passages 9 ^h + m				Interval between Passages 9 ^h + m				Interval between Passages 9 ^h + m
1906. d	h	m		1906. d	h	m		1906. d	h	m	
Jan. 21	15	49.17		Apr. 9	14	8.08		Sept. 4	15	46.61	
23	17	1.97		11	15	21.42		6	16	59.35	
25	18	14.79		13	16	34.76		8	18	12.08	
27	19	27.64		15	17	48.10		10	19	24.79	
29	20	40.51		17	19	1.44		12	20	37.49	
31	21	53.40		19	20	14.78		14	21	50.17	50.54
Feb. 2	23	6.32	50.59	21	21	28.13		16	23	2.83	
5	0	19.26		23	22	41.48		19	0	15.48	
7	1	32.22		25	23	54.83	50.67	21	1	28.11	
9	2	45.21		28	1	8.18		23	2	40.72	
11	3	58.22		30	2	21.53		25	3	53.31	
13	5	11.24		May 2	3	34.88		27	5	5.89	
15	6	24.28		4	4	48.23		29	6	18.45	
17	7	37.34		July 15	9	23.14	50.62	Oct. 1	7	30.99	
19	8	50.42		17	10	36.25		3	8	43.51	
21	10	3.52		19	11	49.34		5	9	56.01	50.50
23	11	16.63	50.62	21	13	2.41		7	11	8.50	
25	12	29.76		23	14	15.47		9	12	20.97	
27	13	42.91		25	15	28.52		11	13	33.42	
Mar. 1	14	56.07		27	16	41.56		13	14	45.85	
3	16	9.24		29	17	54.59		15	15	58.27	
5	17	22.42		31	19	7.60		17	17	10.66	
7	18	35.61		Aug. 2	20	20.60		19	18	23.04	
9	19	48.82		4	21	33.58	50.59	21	19	35.41	
11	21	2.05		6	22	46.55		23	20	47.76	
13	22	15.29		8	23	59.51		25	22	0.09	50.46
15	23	28.53	50.65	11	1	12.45		27	23	12.40	
18	0	41.78		13	2	25.38		30	0	24.70	
20	1	55.04		15	3	38.29		Nov. 1	1	36.98	
22	3	8.31		17	4	51.18		3	2	49.23	
24	4	21.60		19	6	4.07		5	4	1.46	
26	5	34.89		21	7	16.94		7	5	13.68	
28	6	48.18		23	8	29.80		9	6	25.89	
30	8	1.48		25	9	42.64	50.57	11	7	38.08	
Apr. 1	9	14.79		27	10	55.46		13	8	50.26	
3	10	28.11		29	12	8.27		15	10	2.42	50.43
5	11	41.43	50.66	31	13	21.07		17	11	14.56	
7	12	54.75		Sept. 2	14	33.85		19	12	26.68	

SYSTEM I.

1906. d h m	Interval between Passages 9 ^h + m	1906. d h m	Interval between Passages 9 ^h + m	1906. d h m	Interval between Passages 9 ^h + m
Nov. 21 13 38.80		Dec. 5 22 3.32	50.43	Dec. 20 6 27.54	
23 14 50.90		7 23 15.37		22 7 39.57	
25 16 2.99		10 0 27.41		24 8 51.61	
27 17 15.07		12 1 39.44		26 10 3.65	50.43
29 18 27.14		14 2 51.46		28 11 15.71	
Dec. 1 19 39.21		16 4 3.48		30 12 27.78	
3 20 51.27		18 5 15.51		1907. Jan. 1 13 39.85	50.42

SYSTEM II.

1905. d h m	m	1905. d h m	m	1905. d h m	m
June 8 1 49.78	55.81	Aug. 4 23 57.10		Oct. 1 21 51.73	
10 3 28.78		7 1 35.70		3 23 29.86	55.63
12 5 7.78		9 3 14.30		6 1 7.96	
14 6 46.77		11 4 52.89		8 2 46.06	
16 8 25.74		13 6 31.45		10 4 24.14	
18 10 4.70	55.79	15 8 10.01		12 6 2.19	
20 11 43.65		17 9 48.54		14 7 40.24	
22 13 22.58		19 11 27.05		16 9 18.26	
24 15 1.50		21 13 5.55		18 10 56.28	
26 16 40.41		23 14 44.03	55.70	20 12 34.28	
28 18 19.31	55.78	25 16 22.48		22 14 12.27	
30 19 58.20		27 18 0.93		24 15 50.25	55.60
July 2 21 37.07		29 19 39.36		26 17 28.21	
4 23 15.93		31 21 17.77		28 19 6.16	
7 0 54.77		Sept. 2 22 56.16		30 20 44.09	
9 2 33.59		5 0 34.52		Nov. 1 22 22.03	
11 4 12.41		7 2 12.87		3 23 59.96	
13 5 51.32	55.76	9 3 51.20		6 1 37.89	
15 7 30.10		11 5 29.52		8 3 15.82	
17 9 8.87		13 7 7.82	55.66	10 4 53.73	
19 10 47.63		15 8 46.09		12 6 31.65	
21 12 26.36		17 10 24.36		14 8 9.56	55.59
23 14 5.08		19 12 2.60		16 9 47.48	
25 15 43.79		21 13 40.83		18 11 25.40	
27 17 22.48		23 15 19.05		20 13 3.32	
29 19 1.16		25 16 57.24		22 14 41.26	
31 20 39.82		27 18 35.42		24 16 19.19	
Aug. 2 22 18.47	55.73	29 20 13.59		26 17 57.14	

SYSTEM II.

1905. d h m	Interval between Passages 9 ^h + m	1906. d h m	Interval between Passages 9 ^h + m	1906. d h m	Interval between Passages 9 ^h + m
Nov. 28 19 35.10		Feb. 13 8 17.38		Apr. 30 21 25.90	
30 21 13.06		15 9 56.32		May 2 23 5.16	
Dec. 2 22 51.03		17 11 35.27		5 0 44.41	55.85
5 0 29.00	55.61	19 13 14.24			
7 2 7.03		21 14 53.23		July 15 4 47.64	55.80
9 3 45.09		23 16 32.24		17 6 26.64	
11 5 23.17		25 18 11.26	55.80	19 8 5.62	
13 7 1.28		27 19 50.30		21 9 44.59	
15 8 39.40		Mar. 1 21 29.36		23 11 23.54	
17 10 17.55		3 23 8.43		25 13 2.48	
19 11 55.71		6 0 47.52		27 14 41.42	
21 13 33.91		8 2 26.61		29 16 20.34	
23 15 12.14		10 4 5.70		31 17 59.24	
25 16 50.39	55.66	12 5 44.81		Aug. 2 19 38.13	
27 18 28.68		14 7 23.94		4 21 17.01	55.77
29 20 7.00		16 9 3.09		6 22 55.87	
31 21 45.35		18 10 42.24	55.83	9 0 34.73	
1906. Jan. 2 23 23.72		20 12 21.40		11 2 13.56	
5 1 2.12		22 14 0.57		13 3 52.38	
7 2 40.54		24 15 39.74		15 5 31.19	
9 4 19.00		26 17 18.93		17 7 9.97	
11 5 57.51		28 18 58.13		19 8 48.75	
13 7 36.05		30 20 37.32		21 10 27.52	
15 9 14.62	55.72	Apr. 1 22 16.53		23 12 6.27	
17 10 53.22		3 23 55.75		25 13 45.00	55.75
19 12 31.85		6 1 34.97		27 15 23.72	
21 14 10.52		8 3 14.19	55.84	29 17 2.42	
23 15 49.22		10 4 53.42		31 18 41.11	
25 17 27.93		12 6 32.66		Sept. 2 20 19.78	
27 19 6.66		14 8 11.90		4 21 58.43	
29 20 45.42		16 9 51.15		6 23 37.06	
31 22 24.21		18 11 30.40		9 1 15.68	
Feb. 3 0 3.01		20 13 9.65		11 2 54.27	
5 1 41.84	55.77	22 14 48.89		13 4 32.86	
7 3 20.69		24 16 28.14		15 6 11.43	55.72
9 4 59.56		26 18 7.40		17 7 49.98	
11 6 38.46		28 19 46.65	55.85	19 9 28.52	

SYSTEM II.

				Interval between Passages $9^h +$ m					Interval between Passages $9^h +$ m					Interval between Passages $9^h +$ m
1906. d	h	m			1906. d	h	m			1906. d	h	m		
Sept. 21	11	7.04			Oct. 26	14	59.07	55.64		Nov. 30	18	46.04		
23	12	45.54			28	16	37.26			Dec. 2	20	23.98		
25	14	24.01			30	18	15.43			4	22	1.92		
27	16	2.48			Nov. 1	19	53.59			6	23	39.84	55.61	
29	17	40.93			3	21	31.72			9	1	17.77		
Oct. 1	19	19.36			5	23	9.83			11	2	55.68		
3	20	57.77			8	0	47.93			13	4	33.59		
5	22	36.16	55.68		10	2	26.01			15	6	11.48		
8	0	14.54			12	4	4.08			17	7	49.38		
10	1	52.89			14	5	42.14			19	9	27.28		
12	3	31.23			16	7	20.18	55.61		21	11	5.19		
14	5	9.55			18	8	58.19			23	12	43.10		
16	6	47.85			20	10	36.19			25	14	21.01		
18	8	26.12			22	12	14.19			27	15	58.91	55.60	
20	10	4.38			24	13	52.17			29	17	36.84		
22	11	42.63			26	15	30.14			31	19	14.79	55.60	
24	13	20.86			28	17	8.09							

The quantities in the ephemeris are to be interpolated directly for the times for which they are required, the equation of light having been already applied.

The position of *Jupiter's* north pole is assumed to be R.A. $17^h 51^m 59^s.13$, N.P.D. $25^\circ 26' 14''.1$, R.A. $17^h 51^m 59^s.60$, N.P.D. $25^\circ 26' 15''.1$ at the beginning of 1905, 1907 respectively.

P denotes the position-angle of the northern extremity of *Jupiter's* axis, reckoned eastward from the northernmost point of the disc.

$L-O + 180^\circ$, $A-O + 180^\circ$ are the jovicentric right ascensions of the Earth and Sun respectively, reckoned in the plane of the planet's equator from O, the point of the vernal equinox of *Jupiter's* northern hemisphere; B B are the jovicentric declinations of the Earth and Sun above the planet's equator.

The adopted values of the diameters at distance 5.20 are: Equatorial $38''.419$; Polar $35''.945$.

The assumed time for light to traverse the unit distance is $498^s.92$, this being the same value as that used by Mr. Marth.

d denotes the jovicentric angle between the Earth and Sun.

Q denotes the position-angle of the point of greatest defect of illumination, and is reckoned eastward from the northernmost point of the disc. It also gives the position-angle of the shadows of the satellites measured from the satellites themselves.

If we call B'' the jovigraphical latitude of the centre of the disc, then $B'' = \left(\frac{a}{b}\right)^2 B = \left(\frac{15.53}{14.53}\right)^2 B$.

The longitudes of *Jupiter's* central meridian are computed with unaltered values of the rates of rotation and of the zero meridians in the two adopted systems. The addition of the "Corr. for Phase" gives the longitudes of the meridians which bisect the illuminated disc.

The sidereal periods of rotation corresponding to the two adopted systems are $9^h 50^m 30^s.004$, $9^h 55^m 40^s.632$.

Every fifth transit of each zero meridian across the centre of the illuminated disc is given; any intermediate transit may be found by applying once or twice the interval between successive transits, this interval being also tabulated.

The continuation of this ephemeris is given in the *Nautical Almanac* for 1907, the same elements being used there except the equatorial and polar diameters and the light-time, which are altered to accord with those used elsewhere in the *Nautical Almanac*.

There will be a transit of the Earth across the Sun's disc as seen from *Jupiter* on 1906 December 27. For an observer at *Jupiter's* centre the Earth's centre would enter on the Sun's disc Dec. 27^d 21^h 49^m G.M.T., and would leave the Sun's disc Dec. 28^d 8^h 33^m G.M.T.; the least distance of centres would be Dec. 28^d 3^h 11^m, when the Earth's centre would be 67''·5 north of the Sun's centre; the Sun's semi-diameter as seen from *Jupiter* would be 189''·5. The laws of recurrence of these transits were discussed in *Monthly Notices*, vol. lxi. 2, p. 117.

Satellite I. transits the disc of *Jupiter*, Dec. 28^d 12^h, and the satellite will then partially occult its own shadow.

Benvenue, 55 Ulundi Road, Blackheath, S.E. :
1905 January 7.

Ephemeris for Physical Observations of Saturn, 1905. 6-7. By A. C. D. Crommelin.

Paris Midnight.	Light Time.	Longitude of Central Meridian.		G.M.T. of Preceding Transit of Zero Meridian.	
		I. 843°750.	II. 812°641.	System I.	System II.
				h m	h m
1905.	m				
May 28	80.60	155°80	77°39	7 24.77	9 33.53
June 2	...	54.85	180.88	10 17.05	6 30.16
7	79.23	313.93	284.40	2 54.92	3 26.47
12	...	213.04	27.95	5 47.09	11 1.13
17	77.91	112.18	131.53	8 39.22	7 57.60
22	...	11.33	235.13	11 31.32	4 54.04
27	76.70	270.51	338.74	4 9.03	1 50.46

Pari- Midnight.	Light Time.	Longitude of Central Meridian.		G.M.T. of Preceding Transit of Zero Meridian.			
		I. 843° 750.	II. 812° 641.	System I.		System II.	
1905.	m			h	m	h	m
July 2	...	169° 71	82° 38	7	1° 04	9	24° 69
7	75° 60	68° 92	186° 03	9	53° 04	6	21° 05
12	...	328° 14	289° 69	2	30° 69	3	17° 39
17	74° 66	227° 36	33° 36	5	22° 68	10	51° 54
22	...	126° 58	137° 03	8	14° 65	7	47° 86
27	73° 93	25° 79	240° 69	11	6° 64	4	44° 20
Aug. 1	...	285° 00	344° 35	3	44° 30	1	40° 52
6	73° 40	184° 19	87° 99	6	36° 34	9	14° 71
11	...	83° 36	191° 61	9	28° 40	6	11° 16
16	73° 11	342° 51	295° 21	2	6° 15	3	8° 14
21	...	241° 62	38° 78	4	58° 33	10	41° 94
26	73° 08	140° 70	142° 32	7	50° 55	7	38° 48
31	...	39° 75	245° 82	10	42° 82	4	35° 08
Sept. 5	73° 28	298° 76	349° 28	3	20° 81	1	31° 76
10	...	197° 71	92° 70	6	13° 24	9	6° 39
15	73° 74	96° 61	196° 06	9	5° 77	6	3° 24
20	...	355° 46	299° 37	1	44° 03	3	0° 19
25	74° 42	254° 26	42° 63	4	36° 72	10	35° 11
30	...	152° 99	145° 83	7	29° 55	7	32° 23
Oct. 5	75° 32	51° 67	248° 97	10	22° 47	4	29° 47
10	...	310° 30	352° 07	3	1° 07	1	26° 76
15	76° 36	208° 87	95° 11	5	54° 17	9	2° 11
20	...	107° 38	198° 08	8	47° 38	5	59° 62
25	77° 57	5° 83	301° 00	11	40° 70	2	57° 24
30	...	264° 23	43° 87	4	19° 67	10	32° 93
Nov. 4	78° 88	162° 58	146° 69	7	13° 15	7	30° 69
9	...	60° 88	249° 46	10	6° 73	4	28° 56
14	80° 24	319° 14	352° 19	2	45° 93	1	26° 49
19	...	217° 35	94° 87	5	39° 66	9	2° 52
24	81° 62	115° 53	197° 52	8	33° 47	6	0° 59
29	...	13° 67	300° 13	11	27° 32	2	58° 73
Dec. 4	82° 97	271° 79	42° 72	4	6° 74	10	34° 95
9	...	169° 89	145° 28	7	0° 67	7	33° 17
14	84° 25	67° 96	247° 82	9	54° 65	4	31° 45
19	...	326° 00	350° 34	2	34° 20	1	29° 74
24	85° 47	224° 04	92° 84	5	28° 22	9	6° 11
29	...	122° 07	195° 34	8	22° 29	6	4° 45
1906.							
Jan. 3	86° 54	20° 09	297° 83	11	16° 36	3	2° 80

Jan. 1905.

Observations of Saturn, 1905-7.

317

Paris Midnight.	Light Time.	Longitude of Central Meridian.		G.M.T. of Preceding Transit of Zero Meridian.			
		I. 843°750.	II. 812°641.	System I.		System II.	
1906.	m			h	m	h	m
Jan. 8	...	278°12	40°32	3	55.93	10	39.19
13	87.46	176°16	142°82	6	49.96	7	37.54
June 2	80.85	212°42	143°68	5	48.14	7	36.07
7	...	111°42	247°13	8	40.51	4	32.77
12	79.45	10°45	350°60	11	32.82	1	29.44
17	...	269°51	94°10	4	10.72	9	3.91
22	78.09	168°60	197°63	7	2.93	6	0.49
27	...	67°72	301°19	9	55.08	2	56.99
July 2	76.79	326°87	44°77	2	32.85	10	31.33
7	...	226°03	148°37	5	24.94	7	27.77
12	75.60	125°21	252°00	8	16.99	4	24.15
17	...	24°42	355°64	11	8.97	1	20.53
22	74°52	283°63	99°30	3	46.65	8	54.71
27	...	182°84	202°96	6	38.64	5	51.05
Aug. 1	73°64	82°06	306°63	9	30.62	2	47.36
6	...	341°28	50°29	2	8.26	10	21.55
11	72°94	240°49	153°95	5	0.27	7	17.88
16	...	139°69	257°60	7	52.28	4	14.23
21	72°46	38°87	1°23	10	44.32	11	48.47
26	...	298°02	104°84	3	22.08	8	44.89
31	72°24	197°15	208°42	6	14.21	5	41.37
Sept. 5	...	96°26	311°97	9	6.38	2	37.88
10	72°25	355°32	55°49	1	44.28	10	12.33
15	...	254°34	158°97	4	36.60	7	8.97
20	72°51	153°32	262°41	7	29.01	4	5.69
25	...	52°25	5°79	10	21.48	11	40.39
30	73°03	311°12	109°13	2	59.69	8	37.28
Oct. 5	...	209°94	212°41	5	52.36	5	34.27
10	73°77	108°70	315°64	8	45.14	2	31.34
15	...	7°41	58°82	11	38.00	10	6.43
20	74°70	266°06	161°94	4	16.57	7	3.69
25	...	164°66	265°00	7	9.63	4	1.05
30	75°80	63°20	8°00	10	2.79	11	36.47
Nov. 4	...	321°68	110°95	2	41.63	8	34.03
9	77°03	220°10	213°84	5	34.99	5	31.68
14	...	118°48	316°68	8	28.43	2	29.44
19	78°35	16°81	59°48	11	21.96	10	5.24

Paris Midnight.	Light Time.	Longitude of Central Meridian.		G.M.T. of Preceding Transit of Zero Meridian.			
		I. 843° 750.	II. 812° 641.	System I.		System II.	
1906.	m			h	m	h	m
Nov. 24	...	275° 08	162° 23	4	1° 14	7	3° 15
29	79° 72	173° 32	264° 94	6	54° 83	4	1° 12
Dec. 4	...	71° 52	7° 61	9	48° 57	11	37° 17
9	81° 09	329° 69	110° 25	2	27° 92	8	35° 25
14	...	227° 82	212° 85	5	21° 80	5	33° 42
19	82° 43	125° 93	315° 42	8	15° 70	2	31° 64
24	...	24° 01	57° 97	11	9° 67	10	7° 91
29	83° 70	282° 07	160° 51	3	49° 19	7	6° 18
1907.							
Jan. 3	...	180° 12	263° 03	6	43° 20	4	4° 48
8	84° 87	78° 17	5° 54	9	37° 23	11	40° 84
13	...	336° 20	108° 03	2	16° 80	8	39° 19
18	85° 91	234° 22	210° 52	5	10° 85	5	37° 53
23	...	132° 26	313° 02	8	4° 89	2	35° 88
28	86° 80	30° 29	55° 52	10	58° 96	10	12° 26

The rotation times (viz. I. $10^h 14^m \cdot 4$ for equatorial spots, and II. $10^h 37^m \cdot 92$ for Barnard's white spot) and zero meridians have been continued unchanged from the previous ephemeris.

Paris Midnight* is chosen for the first three columns in order to utilise the data of the *Connaissance des Temps* without interpolation. The times of transit of zero meridian are in Greenwich Mean Time. The quantities are to be interpolated for the times for which they are required, the equation of light having been already applied. The small figures between the transits of zero meridian denote the number of intervening rotations, and the following table facilitates the determination of the times of intermediate transits. The argument is found by taking the difference of consecutive values of the longitude of central meridian; in taking the difference it is sufficient to look at the unit and first decimal figure.

Increase of Central Meridian in 5 days.		6 Rota- tions.		5 Rota- tions.		4 Rota- tions.		3 Rota- tions.		2 Rota- tions.		1 Rota- tion.	
	°	h	m	h	m	h	m	h	m	h	m	h	m
System I.	4219° 2	61	26° 01	51	11° 67	40	57° 34	30	43° 00	20	28° 67	10	14° 33
	4218° 8		26° 36		11° 96		57° 57		43° 18		28° 78		14° 39
	4218° 4		26° 71		12° 25		57° 80		43° 35		28° 90		14° 45
	4218° 0		27° 06		12° 55		58° 04		43° 53		29° 02		14° 51
System II.	4063° 7	63	47° 06	53	9° 21	42	31° 37	31	53° 53	21	15° 68	10	37° 84
	4063° 3		47° 43		9° 53		31° 62		53° 72		15° 81		37° 90
	4062° 9		47° 81		9° 84		31° 87		53° 90		15° 94		37° 97
	4062° 5		48° 18		10° 15		32° 12		54° 09		16° 06		38° 03

* That is $9^m 20^s \cdot 9$ before Greenwich Midnight.

The times of transit of zero meridian are given above for the true centre of the disc. If the times are required for the centre of the illuminated disc the following correction should be applied, being subtracted before opposition and added after it.

Days from Opposition.	Correction Min.	Days from Opposition.	Correction Min.	Days from Opposition.	Correction Min.
5	0'00	55	0'19	105	0'26
10	'01	60	'21	110	'24
15	'02	65	'23	115	'23
20	'04	70	'25	120	'21
25	'06	75	'26	125	'19
30	'08	80	'27	130	'17
35	'10	85	'28	135	'15
40	'12	90	'28	140	'12
45	'15	95	'27	145	'10
50	'17	100	'27	150	'08

Erratum in Mr. Franks's Paper.

Page 159, 6th line from bottom, *for* darkness *read* thickness.

